NAA'08: Fourth International Conference on Numerical Analysis and Applications

June 16 - 20, 2008, Lozenetz

University of Rousse
Rousse, Bulgaria

NAA'08

ABSTRACTS

Fourth International Conference on

Numerical Analysis and Applications

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June 16 - 20, 2008, Lozenetz, Bulgaria

Organized by
Department of Mathematics and Informatics
University of Rousse "Angel Kanchev"

Main tracks: Numerical Approximation and Computational Geometry, Numerical Linear Algebra and Numerical Solution of Transcendental Equations, Numerical Methods for Differential Equations, Numerical Modelling, High Performance Scientific Computing, Reliable numerical modelling in science and engineering, Robust Numerical Methods for Multiscale Singular Perturbation Problems.

List of Keynote Speakers who accepted our invitation:

G. Akrivis (Greece), F. C. Chatelin (France), Ch. Christov (USA), I. Farago (Hungary), B. S. Jovanovic (Serbia), N. Kopteva (Ireland), S. Larson (Sweeden), R. Lazarov (USA), V. Makarov (Ukraine), S. Margenov (Bulgaria), P.Matus (Belarus), G. Milovanovic (Serbia), P.Minev(Canada), S. Nicaise (France), E. O'Riordan (Ireland), B. Popov(USA), V. Shaidurov (Russia), G. Shishkin (Russia), P. Vabishchevich (Russia)

Organizing Committee:

Chairmen: Lubin Vulkov
Ivanka Angelova, Juri Kandilarov, Miglena Koleva

Taylor's Decomposition on Two Points for One Dimensional Bratu Problem

Meltem Evrenosoğlu Adıyaman, Sennur Somalı

The boundary problem

$$\Delta y + \lambda e^y = 0,$$

where $y = 0$ on the boundary is often referred to as "the Bratu Problem". It is nonlinear eigenvalue problem with two known bifurcated solutions for $\lambda < \lambda_c$, no solutions for $\lambda > \lambda_c$ and a unique solution when $\lambda = \lambda_c$. In this study, Taylor's Decomposition method is introduced for solving one dimensional Bratu Problem. The numerical scheme is based on the application of the Taylor's decomposition to the first order differential equation system. The technique is illustrated with three numerical examples and the results show that the method converges rapidly and approximates the exact solution very accurately without using small step-size.

A New Method for Computing a Solution of the Cauchy Problem with Polynomial Data for the System of Crystal Optics

Meltem Altunkaynak, Valery Yakhno

A new analytical method for solving an initial value problem (IVP) for the system of crystal optics with polynomial data and a polynomial inhomogeneous term is suggested. The found solution of IVP is a polynomial. Theoretical and computational analysis of polynomial solutions and their comparison with non polynomial solutions corresponding to smooth data are given. The applicability of polynomial solutions for the physical processes is discussed. An implementation of this method has been made by symbolic computations in Maple 10.

Optimal Order FEM for a Coupled Eigenvalue Problem on 2D Overlapping Domains

Andrey Andreev, Milena Racheva

In this paper we present a numerical approach to a nonstandard second-order elliptic eigenvalue problem defined on two overlapping rectangular domains with a nonlocal (integral) boundary condition. Usually, for these class of problems, error estimates are suboptimal. By introducing suitable degrees of freedom and a corresponding interpolation operator we derive optimal order finite element approximation. Numerical results illustrate the efficiency of the proposed method.

New Approach of FEM for Eigenvalue Problems with Non-local Transition Conditions

Andrey Andreev, Milena Racheva

This paper is considered with the finite element method (FEM) for second-order eigenvalue problems on a bounded multi-component domain in the plane. Non-local transition conditions on the interfaces between any two subdomains are imposed. A new finite element approach is proposed, based on much more comprehensible theoretical proofs obtained under lower regularity requirements. The utility of this strategy when superconvergent postprocessing procedure is used as well as the numerical implementation are discussed. Finally, some numerical results are given.

On the Influence of the Air upon Flexure Waves Diffraction in Thin Elastic Plate

Ivan Andronov

When solving problems of flexure waves diffraction by obstacles in thin elastic plates, the influence of air is often neglected. In this paper the correctness of this approach is examined. Using the examples of obstacles of three types (point mass, short cut and small hole) it is discovered that the error of the approximate approach in which the back influence of air on the plate is neglected, can be sufficiently large. In the case of obstacles which do not have holes, the errors can reach approximately 4contains free edges, the errors of approximate approach increase and one can not neglect these errors not only at low frequencies, but also at frequencies close and above coincidence frequency. The errors in the case when the obstacle contains hole of nonzero square appear even more essential.

The cuts and holes in the test examples are described with the use of generalized point models developed in [I.Andronov "Generalized point models in structural mechanics", World Scientific 2002]. The "boundary" conditions corresponding to these models are formulated in the form of expressions interconnecting coefficients in local field asymptotics.

The Numerical Solution of the Cast Billet Heating Problem with Nonlinear and Nonlocal boundary Conditions of the Radiation Type

Viktor Arkhipov, Alexander Glushak and Oiga Versilina

This paper is devoted to the numerical calculation method of the mathematical model of cast rectangular cross section billets heating in metallurgical heating furnaces in view of their mutual heat exchange radiation under the Stefan-Boltsman low.

Let $\Omega = \bigcup_{i=1}^{m} \Omega_i,\ \Omega_i \bigcap \Omega_j = 0,\ \Omega_i = \{M \in R^3 :\ a_{k,j} \le x_k \le b_{k,j},\ k = \overline{1,3}\}$ where Ω_i - are heating billets, $M(x_1, x_2, x_3)$ is variable point of the billet, $\Gamma = \Gamma_1 \bigcup \Gamma_2$, $mes(\Gamma_1 \bigcap \Gamma_2) = 0$ - is the boundary area Ω (billets common surface), ν- the vector of external normal to Γ.

The considering problem is: to find the function $T(M,t)$ - of billets temperature distribution in area $G = \Omega \times \{0 \le t \le t_0\} = \Omega \times W$ satisfying to the equation

$$\rho\frac{\partial T}{\partial t}-\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\lambda(T)\frac{\partial T}{\partial x_i})=0 \quad (1)$$

the initial and boundary conditions:

$$T(M,0)=u_0(M),\ x\in\Omega \quad (2)$$

$$\lambda(T)\frac{\partial T}{\partial \nu}+h(T-T_f)+\chi(T^4-\int\limits_{\Gamma_1}\varphi(M,\xi)T^4(\xi)d\xi)=0,\ \ M\in\Gamma_1 \quad (3)$$

$$\lambda(T)\frac{\partial T}{\partial \nu}+h(T-T_f)=0,\ M\in\Gamma_2,\qquad t\in(0;t_0]; \quad (4)$$

where $\rho,\ \lambda(T),\ h\ \chi$ - factors determining metal internals and heat exchanging conditions on it's surface, T_f - variable temperature of external furnace space, $\varphi(M,\xi)$ - an elementary angel coefficients of billets' mutual radiation.

The problem (1)-(4) is solved by the method of the difference approximation with using the common implicit difference scheme. In definite factors conditions the priori estimates have been got and one-valued solvability of difference problem's (1)-(4) analogue has been determined. The iteration methods of solving and an estimation of their convergence have been supposed. The mixed methods of the decision are considered.

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N 06-08-96312.

Minimal Simplex for IFS Fractal Sets

Elena Babače, Ljubiša Kocić

Fractal sets manipulation and modeling is a difficult task due to their complexity and unpredictability. One of the basic problems is to determine bounds of a fractal set given by some recursive definitions, for example by an Iterated Function System (IFS). Here we propose a method of bounding an IFS-generated fractal set by a simplex that is affinely identical to the standard simplex and is arbitrarily close to the minimal one. First, it will be proved that for a given IFS attractor, such simplex exists and that is unique. Such simplex is then used for definition of an Affine invariant Iterated Function System (AIFS) that then can be used for affine transformation of a given fractal set and for its modeling.

An Efficient Computational Technique For a System of Singularly Perturbed Initial Value Problems

Rajesh Bawa, Vinod Kumar

Many physical processes connected with nonuniform transitions are described by differential equations with large and/or small parameters. If, in a problem arising in this manner, the role of the perturbation is played by the leading terms of the differential operator (or part of them), then the problem is called singular perturbation problem. In particular, systems of singularly perturbed first order ordinary differential equations occur in many areas like control theory, chemical reactor theory etc.

In this paper, we consider following class of system of first order singularly perturbed ordinary differential equations of the type:

$$L_\epsilon u_\epsilon(x) \equiv \begin{cases} (L_\epsilon u_\epsilon)_1(x) = \epsilon D u_{\epsilon,1}(x) + a_{11}u_{\epsilon,1}(x) + a_{12}u_{\epsilon,2}(x) + ... + a_{1n}u_{\epsilon,n}(x) = f_1(x) \\ (L_\epsilon u_\varepsilon)_2(x) = \epsilon D u_{\epsilon,2}(x) + a_{21}u_{\epsilon,1}(x) + a_{22}u_{\epsilon,2}(x) + ... + a_{2n}u_{\epsilon,n}(x) = f_2(x) \\ . \\ . \\ . \\ (L_\epsilon u_\epsilon)_n(x) = \epsilon D u_{\epsilon,n}(x) + a_{n1}u_{\epsilon,n}(x) + a_{n2}u_{\epsilon,1}(x) + ... + a_{nn}u_{\epsilon,n}(x) = f_n(x) \end{cases}$$

$x \in (0,1], u_{\epsilon,j}(0) = u_i^0$ for $i = 1(1)n$, where $u_\epsilon = (u_{\epsilon,1}, u_{\epsilon,2}, ...u_{\epsilon,n})^T$ and $u_\epsilon \in C^{(1)}(\Omega)$. The singular perturbation parameter ϵ satisfies $0 < \epsilon \leq 1$. The functions $a_{ij}, f_i \in C^2(\overline{\Omega}), i, j = 1, 2, ..., n$ satisfies the following inequalities:

$$(i) \quad a_{ii} > \sum_{j=1, j\neq i}^{n} |a_{ij}(x)|, \quad i = 1(1)n$$

$$(ii) \quad a_{ij} < 0, \quad i, j = 1(1)n, i \neq j$$

We propose parameter-uniformly convergent computational technique for a system of singularly perturbed initial value problems, which is applied on an well known piecewise uniform mesh namely, the Shishkin mesh. Numerical experiments carried out on some test problems shows almost second order uniform convergence, confirming the efficiency of the proposed technique.

Model Predictive Control - Numerical Procedures for the Invariant Sets Approximation

H. Benlaoukli, Sorin Oralu

Model Predictive Control (MPC) is an optimization based control technique which applies in a receding horizon manner the solution of an open loop optimal control problem. The feasibility turns out to be a crucial demand in MPC synthesis as long as it represents the main ingredient for the stability of the entire closed loop system. In this context, the positive invariance is an important concept to be exploited for assuring the feasibility at all future instants and by consequence the viability of the control law.

This paper deals with the computational issues encountered in the construction of invariant sets of LTI (Linear Time Invariant) systems. It is shown that explicit MPC laws transform the closed loop dynamics in a piecewise affine (PWA) system. The main contribution is the comparison of two effective numerical procedures for the outer approximations of the maximal positively invariant (MPI) set for such PWA systems. Indeed, when iterative construction procedures are employed especially if no finite-time algorithms exists to construct the exact MPI set, consistent approximations are to be found withing a predefined precision.

The main tools employed here are the polyhedral computations but in order to decrease the computational complexity, the interval analysis procedure and transition graph constructions are used to avoid the treatment of the regions which do not meet the neighboring properties.

the conclusion is that computational geometry plays an important role in providing versatile set theoretic methods in engineering problems involving constraints, bounded-disturbances or construction of domain of attraction for stabilizing compensators.

Finite Difference Solution of a Nonlinear Klein-Gordon Equation with an External Source

G. Berikelashvili, J. Gvazava, O. Jokhadze, S. Kharibegashvili and B. Midodashvili

In this paper, we consider the first Darboux problem for cubic nonlinear Klein-Gordon equation with an external source

$$\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial x^2} + m^2 v + \lambda v^3 = \varphi(x,t), \quad (x,t) \in D_T,$$

$$v(0,t) = 0, \quad v(t,t) = 0, \quad 0 \le t \le T,$$

where $\lambda > 0,\ m \ge 0$ are constants, $D_T := \{(x,t) : 0 < x < t, 0 < t < T\}$.

Stable finite difference scheme is constructed on a four-point stencil, which does not require additional iterations for passing from one level to another.

It is proved, that the finite difference scheme converges with the rate $O(h^2)$, when the exact solution belongs to the Sobolev space W_2^2.

Quartic spline of interpolation with minimal quadratic oscillation

Alexandru Mihai Bica

The quartic spline of interpolation generated by initial conditions is constructed. The initial values corresponding to the first, second and third derivative of spline in the first knot are uniquely determined such that the quadratic oscillation in average of the quartic spline to be minimal (this notion was recently introduced by the author for any spline of interpolation function).

Expressions of solutions of linear partial differential equations using algebraic operators and algebraic convolution

Liepa Bikulčienė, Zenonas Navickas

The most popular differential equations in various theoretical and practical applications are the following ones $D_x^k u = (P(s) D_s^m + Q(t) D_t^n) u$.

Here $\frac{\partial}{\partial x} := D_x, \ldots, \frac{\partial}{\partial t} := D_t$, $P(s)$ and $Q(t)$ - are polynomials in s and t. Therefore, it is expedient to have exact expressions for the solutions of differential equations $u = u(x; s, t)$.

The solutions $p = p(x; s)$ and $q = q(x; t)$ of

$$\left(D_x^k - P(s) D_s^m\right) p = 0, \left(D_x^k - Q(t) D_t^n\right) q = 0$$

with special boundary conditions have the algebraic form:

$$p(x,s) = \sum_{j=0}^{\infty} \lambda_j(s) \frac{x^j}{j!}, q(x,s) = \sum_{j=0}^{\infty} \mu_j(t) \frac{x^j}{j!}.$$

Here polynomials $\lambda_j(s)$ and $\mu_j(t)$ are formed by a corresponding operator algorithm.

Then, dependently on $k, m, n \in N$, after introduction of an algebraic convolution operation $'*'$, solution of the given differential equation is presented in the form:

$$u = u(x; s, t) = p(x, s) * q(x, t).$$

The proposed computational algorithm is constructed using the operator property

$$(P(s) D_s^m)(Q(t) D_t^n) = (Q(t) D_t^n)(P(s) D_s^m)$$

and other identities.

Convergence in $W_2^{2,1}$ norm of FDM for the Parabolic Problem with Concentrated Capacity and Time-Depended Operator

Dejan Bojović , Boško Jovanović

The convergence of difference method for the one-dimensional heat equation with time-depended operator and the coefficient at the time derivative (heat capacity) $c(x) = 1 + K\delta(x - \xi)$, is investigated. Coefficient $c(x)$ represents the magnitude of the heat capacity concentrated at the point $x = \xi$. An abstract operator method is developed for analyzing this equation. Estimate for the rate of the convergence in special discrete energetic $W_2^{2,1}$ Sobolev norm, compatible with the smoothness of the solution is obtained.

The Transmission Problem For Elliptic Second Order Equations In A Conical Domain

Mihail Borsuk

The present talk is a survey of our last results. Let $G \subset \mathbb{R}^n$, $n \geq 2$ be a bounded domain with boundary ∂G that is a smooth surface everywhere except at the origin $\mathcal{O} \in \partial G$ and near the point $\mathcal{O}$ it is a conical surface with vertex at $\mathcal{O}$ and the opening ω_0. We assume that $G = \bigcup_{i=1}^{N} G_i$ is divided into $N \geq 2$ subdomains $G_i,\ i = 1, \dots, N$ by $(N-1)$ hyper-planes $\Sigma_k,\ k = 1, \dots, N-1$, (by Σ_0 for the case $N = 2$), where $\mathcal{O}$ belongs to every $\overline{\Sigma_k}$ and $G_i \cap G_j = \emptyset,\ i \neq j$. We consider estimates of the weak solutions to the following elliptic transmission problems near conical boundary point:

- problem (LN) for the Laplace operator with N different media and mixed boundary condition

$$\begin{cases} \mathcal{L}_i[u] \equiv a_i \triangle u_i - p_i u_i(x) = f_i(x), & x \in G_i,\ i = 1, \dots, N; \\ [u]_{\Sigma_k} = 0, \quad \mathcal{S}_k[u] \equiv \left[a \frac{\partial u}{\partial n_k}\right]_{\Sigma_k} + \frac{1}{|x|}\beta_k(\omega)u(x) = h_k(x), & x \in \Sigma_k, \\ & k = 1, \dots, N-1; \\ \mathcal{B}[u] \equiv \alpha(x) a \frac{\partial u}{\partial \vec{n}} + \frac{1}{|x|}\gamma(\omega)u(x) = g(x), & x \in \partial G \setminus \mathcal{O}, \end{cases}$$

  where $\text{ø} = \frac{x}{|x|},\ a_i > 0,\ p_i \geq 0, \quad (i = 1, \dots, N)$ are constants; $\alpha(x) = \begin{cases} 0, & \text{if } x \in \mathcal{D} \\ 1, & \text{if } x \notin \mathcal{D}, \end{cases}$ and $\mathcal{D} \subseteq \partial G$ is the part of the boundary ∂G where we consider the Dirichlet boundary condition;

- problem (L) for linear equations

$$\begin{cases} \mathcal{L}[u] \equiv \frac{\partial}{\partial x_i}\left(a^{ij}(x)u_{x_j}\right) + a^i(x)u_{x_i} + a(x)u = f(x), x \in G \setminus \Sigma_0; \\ [u]_{\Sigma_0} = 0, \quad \mathcal{S}[u] \equiv \left[\frac{\partial u}{\partial \nu}\right]_{\Sigma_0} + \frac{\beta(\omega)}{|x|}u(x) = h(x), \qquad x \in \Sigma_0; \\ \mathcal{B}[u] \equiv \frac{\partial u}{\partial \nu} + \frac{\gamma(\omega)}{|x|}u = g(x), \qquad x \in \partial G \setminus \mathcal{O}; \end{cases}$$

- problem (WL) for weak nonlinear equations

$$\begin{cases} -\dfrac{d}{dx_i}\left(|u|^q a^{ij}(x)u_{x_j}\right) + b(x,u,\nabla u) = 0, \quad q \geq 0, x \in G \setminus \Sigma_0; \\ [u]_{\Sigma_0} = 0, \quad \mathcal{S}[u] \equiv \left[\frac{\partial u}{\partial \nu}\right]_{\Sigma_0} + \frac{\beta(\omega)}{|x|}u|u|^q = h(x,u), \quad x \in \Sigma_0; \\ \mathcal{B}[u] \equiv \frac{\partial u}{\partial \nu} + \frac{\gamma(\omega)}{|x|}u|u|^q = g(x,u), \quad x \in \partial G \setminus \mathcal{O}; \end{cases}$$

- problem (QL) for general elliptic divergence quasi-linear equations

$$\begin{cases} -\dfrac{d}{dx_i}a_i(x,u,\nabla u) + b(x,u,\nabla u) = 0, \quad x \in G \setminus \Sigma_0; \\ [u]_{\Sigma_0} = 0, \quad \mathcal{S}[u] \equiv \left[\frac{\partial u}{\partial \nu}\right]_{\Sigma_0} + \frac{1}{|x|^{m-1}}\sigma\left(\frac{x}{|x|}\right) u \cdot |u|^{q+m-2} = h(x,u), \quad x \in \Sigma_0; \\ \mathcal{B}[u] \equiv \frac{\partial u}{\partial \nu} + \frac{1}{|x|^{m-1}}\gamma\left(\frac{x}{|x|}\right) u \cdot |u|^{q+m-2} = g(x,u), \quad x \in \partial G \setminus \mathcal{O}. \end{cases}$$

We establish estimates of the type $u(x) = O(|x|^\alpha)$. For problems (LN), (L) and (WL) exponent α is **the best possible**. A principal new feature of our work is the consideration of estimates of weak solutions for *general divergence quasi-linear* elliptic second-order equations in $n-$ *dimensional conic* domains and for *linear* elliptic equations with *minimal smooth coefficients*. Our examples demonstrate this fact.

Multilevel Splitting of Weighted Graph-Laplacian Arising in Non-conforming Mixed FEM Elliptic Problems

Petia Boyanova, Svetozar Margenov

We consider the following mixed elliptic problem:

$$\begin{aligned} \mathbf{u} + \nabla p &= \mathbf{f} && \text{in } \bar{\Omega}, \\ \nabla \cdot \mathbf{u} &= 0 && \text{in } \Omega, \\ \mathbf{u} \cdot \mathbf{n} &= 0 && \text{on } \partial\Omega \end{aligned}$$

It has to be solved as a part of a projection algorithm for unsteady Navier-Stokes equations. The use of Crouzeix-Raviart non-conforming elements for the velocities $\mathbf{u}$ and piece-wise constant approximation of the pressure p provides a stable discretization with respect to the Reynolds number. Then, the Crouzeix-Raviart mass matrix is diagonal, and the velocity unknowns can be eliminated locally. The reduced matrix for the pressure is referred to as weighted graph-Laplacian.

In this paper we study the construction of optimal order preconditioners based on algebraic multilevel iterations (AMLI). The weighted graph-Laplacian for the model 2-D problem is considered. We assume that the finest triangulation is obtained after recursive uniform refinement of a given coarse mesh. The introduced hierarchical splitting is the first important contribution of this article. The proposed construction allows for a local analysis of the constant in the strengthened Cauchy-Bunyakowski-Schwarz (CBS) inequality. This is an important characteristic of the splitting and is associated with the angle between the two hierarchical FEM subspaces. The estimates of the convergence rate and the computational cost at each iteration show that the related AMLI algorithm with acceleration polynomial of degree two or three is of optimal complexity.

Some contributions of Homotopic Deviation to the theory of matrix pencils

F. Chatelin, Morad Ahmadnasab

Let $A, E \in \mathbb{C}^{n\times n}$ be two given matrices, where rank $E = r < n$. The matrix E is written in the form (derived from SVD) $E = UV^H$ where $U, V \in \mathbb{C}^{n\times r}$ have rank r. 0 is an eigenvalue of E with algebraic (resp. geometric) multiplicity m ($g = n - r \leq m$). $\sigma(A)$ is the spectrum of A.

We consider the pencil $P_z = (A - zI) + tE$, defined for $t \in \hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ and depending on the complex parameter $z \in \mathbb{C}$. We analyze how its structure evolves as the parameter z varies, by means of conceptual tools borrowed from Homotopic Deviation theory which is fully original. The pencil P_z is regular for z in the resolvent set $\mathbb{C}\backslash\sigma(A)$, or if z is an evolving eigenvalue. The Weierstrass structure for P_z is specified by the eigenvalues of the communication matrix

$$M_z = V^H(zI - A)^{-1}U, \quad E = UV^H.$$

This matrix of order r, together with the augmented pencil of order $n + r$

$$\begin{bmatrix} zI - A & -U \\ V^H & 0 \end{bmatrix},$$

play an essential role. In particular, an important feature is that, because t varies in $\hat{\mathbb{C}}$, we can look at what happens in the limit when $|t| \to \infty$. New notions which complement the purely algebraic theory of Weierstrass are therefore derived. As an example with $z = 0$, the structure of the pencil $A + tE$ is determined by Homotopic Deviation when $0 \notin \sigma(A)$. Numerical illustrations are provided.

Stability and Bifurcation of the Magnetic Flux bound states in Josephson Junctions

Ivan Christov, Stefka Dimova and Todor Boyadjiev

The static distributions of the magnetic flux in Josepfson junctions are investigated numerically. To solve the nonlinear boundary value problem an iterative algorithm, based on the Continuous analog of Newton method is constructed. The linearized problems at every iteration step are solved by the Galerkin finite element method.

In order to study the stability of possible distributions a Sturm-Liouville problem is generated whose minimal eigenvalue determines the stability of the solution. A minimal eigenvalue equal to zero means a bifurcation of the corresponding solution. The subspace iteration method is used to find the smallest eigenvalues and the corresponding eigenvectors. The main physical characteristics (energy, number of fluxons, full magnetic flux and so on) are computed. The dependence of these characteristics on the geometric parameters of the junction is investigated and new interesting results are found.

Orthogonal systems with respect to the complex Borel measures

Gradimir V. Milovanović, Aleksandar S. Cvetković

It is well-known and established result that in the case of positive Borel measures Gaussian quadrature rules exist and are unique and exact on the suitable polynomial spaces, for the complex Borel measures this is not the case. In this lecture we address the questions connected to the existence and construction of Gaussian quadrature rules for the complex Borel measures supported on the real line.

We present general results connected with the theory of the respective Jacobi operator, and questions of convergence of Padé approximation in the regions of the complex plane. Results about the convergence of Padé approximation can be used to give general results about the convergence of the Gaussian quadrature rule. We present several results connected with the existence of orthogonal polynomials with respect to the families of measures.

Finally, we present several families of the Borel measures on the real line which admit Gaussian quadrature rules and give properties of the computation of the Gaussian quadrature rules with respect to these measures.

On a Class of Almost Orthogonal Polynomials

Bratislav Danković, Predrag M. Rajković

It is known that orthogonal polynomials are useful tool in technical sciences. Here we will emphasize their role in signal approximation and design of the electronic systems which generate the orthogonal signals. One class of those systems are orthogonal systems. However, since the components of those systems can not be made quite exactly, the polynomials which are generated by these systems are not quite orthogonal, but rather *almost orthogonal*. The measure of nearness between obtained and regular orthogonal polynomials depends on the exactness of the manufacturing of the components. Until now, some classes of almost orthogonal functions are defined and investigated. In this paper, we define a new class almost orthogonal polynomials which can be used successfully for modelling of electronic systems which generate orthonormal basis. They are very suitable for analysis and synthesis of imperfect technical systems which are projected to generate orthogonal polynomials, but in the reality generate almost orthogonal polynomials.

Tensor Product $q-$Bernstein Bézier Patches

Çetin Dişibüyük, Halil Oruç

In this work we define a new de Casteljau type algorithm, which is in barycentric form, for the $q-$Bernstein Bézier curves. We express the intermediate points of the algorithm explicitly in two ways. Furthermore we define tensor product patches, based on this algorithm, depending on two parameters. Degree elevation procedure for the tensor product patch is studied. Finally, the matrix representation of tensor product patch is given and we find the transformation matrix between classical tensor product Bézier patch and tensor product $q-$Bernstein Bézier patch.

Numerical Experiments for Reaction-Diffusion Equations Using Spectral Methods

Gabriel Dimitriu, Răzvan Ştefănescu

Reaction-diffusion equations are frequently encountered in mathematical biology, ecology, physics and chemistry, and many mathematical models exist for special cases. This type of equations leads to many interesting phenomena, such as, pattern formation far from equilibrium, pulse splitting and shedding, Turing structures, reactions and competitions in excitable systems, nonlinear waves such as solitons or spiral waves and spatio-temporal chaos.

The efficient and accurate simulation of such systems, however, represent a difficult task. This is because they couple a stiff diffusion term with a (typically) strongly nonlinear reaction term. When discretised this leads to large systems of strongly nonlinear, stiff ODEs. Moreover, finite-difference methods can sometimes lead to spurious, that is nonphysical, solutions.

In this study we focus on the numerical approach of several reaction-diffusion equations by using a spectral technique. The numerical method is spectrally accurate in space, fourth order accurate in time, and uses periodic boundary conditions. A numerical stabilisation technique is also applied by computing certain coefficients with the help of integrals around contours in the complex plane. Numerical simulations in 1D and 2D on a range of reaction-diffusion problems using this method are presented.

Diaphony of Uniform Samples over Hemisphere and Sphere

I. Dimov, S. Stoilova and N. Mitev

The sampling of certain solid angle is a fundamental operation in realistic image synthesis, where the rendering equation describing the light propagation in closed domains is solved. In this work we consider the problem for generation of uniformly distributed random samples over hemisphere and sphere. Using two algorithms we obtain samples in orthogonal spherical triangle and spherical quadrangle. Our aim is to prove uniform distribution of the obtained samples. The importance of the uniformly distributed samples is determined by the effectiveness of the algorithms for numerical solution of the rendering equation. We use numerical characteristic for uniform distribution of points, called diaphony. The diaphony of these samples is calculated numerically. Analysis and comparison of the obtained results are made.

Modeling of a vertical cavity surface emitting laser containing a multi-QW heterostructure

N. Elkin, A. Napartovich, V. Troshchieva and D. Vysotsky

A vertical cavity surface emitting laser (VCSEL) is an object of numerical study in the presented paper.

The linear non-hermitian eigenvalue problem arises in the first stage when we consider a "cold" cavity and neglect change of material characteristics induced by electromagnetic field. The round-trip operator technique and Krylov subspace methods were used for determination of eigenfunctions, which represent intra-cavity wave field distributions. Corresponding complex eigenvalues determine the wavelength shifts relative to reference value and threshold gains.

The next stage of study relates to a case of a loaded cavity when self-consistent solving of a wave field equation and material equations is required. The eigenvalue problem for a non-linear operator must be solved to find the lasing electromagnetic field spatial profile and its frequency. The gain element of a typical VCSEL device comprises several quantum wells (QW). The charge carriers distributions in each of QW obey non-linear diffusion equation. The round-trip operator is a non-linear operator in this case, and its evaluation needs an iteration procedure. We propose the iteration procedure, which is applicable for a set of QW of any size and has computational costs growing linearly with number of QW.

The computational procedures and results of calculations for a cylindrical VCSEL will be reported.

Maximum Principles for the Parabolic Problems

István Faragó

When we construct mathematical and/or numerical models in order to model or solve a real-life problem, these models should have different qualitative properties, which typically arise from some basic principles of the modelled phenomena. In other words, it is important to preserve the characteristic properties of the original process, i.e., the models have to possess the equivalents of these properties. E.g., many processes, varying in time, have such properties as the monotonicity, the non-negativity preservation and the maximum principles. We will examine these qualitative properties in this talk. We note that, even if we consider mostly simple problems, like the so-called heat equation, they can be viewed as a sub-problem, obtained by using the operator splitting for a complex reaction-diffusion-advection equation. Hence, for such a simple problem, the conditions of the preservation of the main qualitative properties of the continuous problem plays an important role, too.

We focus our attention to the investigation of the maximum principles which means the following. we give lower and upper bounds for the distribution of temperature in the body. These bounds are defined by the (known) values of the temperature at the boundary, the initial state and source. One of the simplest form of them states that, if there are no heat sources and sinks present inside the body, then the maximum temperature appears also on the boundary of the body or in the initial state.

We investigate the heat equation in 3D in more details and formulate those conditions under which the discrete maximum principle holds. We illustrate our results by some numerical results.

Memetic Simulated Annealing for GPS Surveying Problem

Stefka Fidanova

A Global Positioning System (GPS) is a satellite-based radio-navigation system that permits land, sea, and airborne users to determine their three dimensional position, velocity, and time. The service is always available at any time and under any weather condition. In addition, satellite navigation systems have an impact in geoscience, in particular on surveying work in quick and effective determining positions and changes in positions networks. The most widely known space systems are: the American NAVSTAR global positioning system, the Russian GLObal Navigation Satellite System (GLONASS), and the forthcoming European satellite navigation system (GALILEO).

In a GPS survey, a number of receivers are placed at stations to be coordinated by setting sessions between these stations. A session is the time period during which two or more receivers simultaneously receive satellite signals for a fixed duration. After the observation, the receivers are moved to other stations for further measurements. This process is repeated until all sessions are measured. The objective of the survey problem is thus to schedule the session observations in order to minimize the cost (time) of the complete process.

Solving this large problem to optimality requires a very high computational time. Therefore, new methods are needed to provide near-optimal solutions for large networks within acceptable amount of computational effort. These techniques are usually based on structured metaheuristics. In this paper several Simulated Annealing (SA) algorithms, are introduced and applied on test problems with different dimension.

On weakening conditions for discrete maximum principles for linear finite element schemes

Antti Hannukainen, Sergey Korotov and Tomáš Vejchodský

Many second order (both linear and nonlinear) elliptic and parabolic problems satisfy the maximum principle. Besides the theoretical importance, the maximum principle mirrors the natural property of the modelled physics. For example, in the heat conduction problem, the maximum principle guarantees the nonnegativity of the obtained temperature. Similarly, the maximum principle is important for modeling of the other naturally nonnegative quantities, like concentration, density, etc.

In this contribution, we concentrate on the lowest-order (piecewise linear) finite element approximations of the second order linear elliptic problems. The known theoretical results suggest certain limitations on the shape and size of the finite element meshes to guarantee the DMP. However, the numerical experiments we will present indicate that the DMP is valid even beyond these theoretical limitations.

Perhaps surprisingly, if we compare the set of meshes, where the DMP is guaranteed by the theory with the set of meshes, where the DMP is confirmed computationally, we find out that the theory does not cover substantial part of meshes yielding the DMP. This indicates that the extension of the known DMP results is possible and creates a challenge for the further theoretical research.

Some Specific Qualitative Properties of the One-Dimensional Heat Conduction Equation

Róbert Horváth

The preservation of the qualitative properties of the heat conduction process to the numerical solution of the heat conduction equation is an important task in numerical modeling. Some known and thoroughly investigated properties are the maximum-minimum principles, the nonnegativity preservation, the monotonicity conservation, sign-stability and the maximum norm contractivity property. In our present paper, we investigate three specific qualitative properties for both the finite difference and the finite element solution of the one-dimensional heat-conduction equation:

- the shape conservation,
- the smoothness increase, and
- the monotone decrease in coordinates.

In the first part of the paper, we show that the above properties are valid for the solution of the heat conduction equation. Then, we define the equivalents of the properties for one-step vector iterations and prove the necessary and sufficient conditions of the validity of the properties. After writing the numerical solution of the heat equation into a vector iteration form, the linear algebraic results can be applied directly. Our main result is that when the vector iteration is convergent and it preserves the nonnegativity then it possesses the above listed three qualitative properties as well.

Modelling Fiscal Policy and Welfare in a Currency Board (a Dynamic General Equilibrium Approach

Iordan Iordanov, Andrey Vassilev

We develop a dynamic general equilibrium model for a small open economy operating under a currency board arrangement, similar to the one presented in [1]. Using different numerical simulations, we study the welfare properties of equilibria in the model and analyze welfare-maximizing fiscal rules.

References

[1] Iordan Iordanov, Andrey Vassilev (2008), A Small Open Economy Model with a Currency Board Feature: the Case of Bulgaria. Bulgarian National Bank Discussion Paper DP/63/2008.

About a Spectral Problem Containing Dirac Distribution

Boško S. Jovanović, Irena M. Jovanović

We consider an eigenvalue transition problem in two disjoint intervals. In a special case, the problem is transformed into spectral problem in canonical interval $(-1,1)$ with conjugation conditions at the midpoint $x=0$:

$$-y''(x) = \lambda y(x), \quad x \in (-1,0) \cup (0,1),$$

$$[y']_{x=0} = 0, \quad [y]_{x=0} = y'(0),$$

$$y(-1) = y(1) = 0.$$

Using Dirac's distribution the problem can be rewritten as:

$$-\big((1-\delta(x))y'\big)' = \lambda y(x), \quad x \in (-1,1),$$

$$y(-1) = y(1) = 0.$$

Eigenvalues and corresponding eigenfunctions are obtained.

Numerical Approximation of Interface and Transmission Problems

Bosko Jovanovic, Lubin Vulkov

The transfer of energy and mass is fundamental in many biological, chemical, environmental and industrial processes. The basic transport mechanisms in such processes are diffusion (or dispersion) and bulk flow. Here we focus on diffusion in domains with layers.

Layers with material properties which significantly differ from those of the surrounding medium appear in a variety of applications. The layer may have a structural role (as in the case of glue), a thermal role (as in the case of thermal insulator), an electromagnetic or optical role, etc.

In the case of thin layers such problems can be modelled by partial differential equations whose input data and the solutions have discontinuities across one or several interfaces (which have lower dimension than the domain where the problem is defined). Standard numerical methods designed for the problems with smooth solutions do not work efficiently in the case of interface problems.

In the case of thick layers in an analogous way one obtains so called transmission problems, whose solutions are defined in two (or more) disjoint domains. For example, such a situation occurs when the solution in the intermediate region is known, or can be determined from a simpler equation. The effect of the intermediate region can be modelled by means of nonlocal jump conditions.

In this paper some model examples of interface and transmission problems are presented and the corresponding numerical methods for their solution are proposed and investigated.

The Weierstrass Canonical Form of a Regular Matrix Pencil: Numerical Issues and Computational Techniques

Grigorios Kalogeropoulos, Marilena Mitrouli, Athanasios Pantelous and Dimitrios Triantafyllou

Matrix pencils are naturally associated with differential (difference) systems of the type $S(F,G)$: $F\dot{\underline{x}} = G\underline{x}(t)$ $(F\underline{x}_{k+1} = G\underline{x}_k)$, $F, G \in C^{n\times n}$, with $detF = 0$, which in turn describe a variety of problems for the descriptor system theory and constitute a more general class than linear state systems. For the treatment of the above systems, the transformation of the original pencil to a canonical form is required.

Thus, in the present paper, we study the derivation of the Weierstrass Canonical Form (WCF) of a regular matrix pencil. In order to compute the WCF, we use two important computational tools: a) the QZ algorithm to specify the required root range of the pencil and b) the updating technique to compute the index of annihilation. The proposed updating technique takes advantages of the already computed rank of the sequences of matrices that appears during our procedure reducing significantly the required floating-point operations.

The algorithm is implemented in a numerical stable manner, giving efficient results. Error analysis and the required complexity of the algorithm are included.

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A Coupling Interface Method for a Parabolic-Elliptic Nonlinear Problem

Juri Kandilarov

A coupling interface method (CIM) on uniform cartesian grid for solving parabolic-elliptic problem is proposed. The domain Ω is divided in two disjoint subregions Ω^- and Ω^+. On Ω^- an elliptic equation with nonlinear term and on Ω^+ - parabolic equation are considered. On the interface Γ the standard transition conditions are stated: the jumps of the solution and the flux are zero. Such kind of problems arise in some fermentation reactors and enzyme reactors in food industry or pharmacy. The approximation consists in two steps: approximation in space, using CIM1 or CIM2 on Cartesian grids and approximation in time, using implicit Euler method. A method of upper and down solutions is used to deal with the nonlinearity. Convergence of first order is numerically confirmed.

Numerical Method for an Elliptic Problem with the Concentrated Source in a Strip

Olga Kharina

We consider questions of the numerical solution of an elliptic problem with the concentrated source in a strip. Using method of lines an elliptic problem is reduced to the system of ordinary differential equations on an infinite interval. For the numerical solution we reduce the problem to a finite interval. We pick out the sets of solutions of equation, satisfying the limit conditions at infinity, by two systems of differential equations of the first order. The coefficients of these systems are solutions of the singular Cauchy problems for the matrix Riccati equations. Stability of the received boundary value problem on the finite interval to perturbation of coefficients is investigated. To get a difference scheme with the property of an uniform convergence, we use piecewise uniform mesh. Numerical experiments have a place.

Self-affine Fractals Generated by Nonlinear Systems

Ljubiša Kocić, Sonja Gegovska-Zajkova and Elena Babače

Nonlinear dynamical systems, defined by a system of ODE's, contain nonlinear terms. Under some conditions these terms can be locally approximated by linear factors, which can be, after discretization transformed in the sequence of (hyperbolic) Iterated Function Systems (IFS) that generates a unique fractal attractor. This attractor reflects the dynamics in the vicinity of the approximated point of the nonlinear system. Here, the IFS is replaced with an associate AIFS (Affine invariant IFS), a kind of IFS that has affine invariant property and permits further manipulating of this fractal attractor.

Mathematical Modelling of Cellular Immune Response to Virus

Mikhail Kolev

A mathematical model describing the interactions between viral infection and cell-mediated immunity is proposed. The model is developed with statistical methods analogous to those of kinetic theory. The interacting individuals are characterized by a microscopic functional state variable. The model is formulated in terms of a bilinear system of partial integro-differential equations. Numerical simulations of the model are presented.

A Two-grid Approximation of an Interface Problem for the Nonlinear Poisson-Boltzmann Equation

Miglena Koleva, Lubin Vulkov

We present a robust and efficient numerical method for solution of an interface problem for a generalization of the Poisson-Boltzmann equation, arising in molecular biophysics. The differential problem is solved by FEM (finite element method) technique on two (coarse and fine) subspaces. The solution of the nonlinear systems of algebraic equations on the fine mesh is reduced to the solution on two small (one linear and one nonlinear) systems on the coarse grid and a large linear one on the fine grid. It is shown, both theoretically and numerically, that the coarse space can be extremely coarse and still achieve asymptotically optimal approximation.

Numerical Solution of an Elliptic Problem with a Non-standard Interface Condition

Natalia Kolkovska

We investigate an elliptic problem with a nonstandard interface condition, which relates the second order tangential derivative and the jump of the normal derivative. This specific condition is usually referred as 'Venttsel' condition.

For the numerical solution of the problem we apply an appropriate finite difference method, which is fast and stable. The discrete Sobolev W_2^1 norm of the error of the method is proved to be of order $m-1$ for solutions from W_2^m, with any m, $1 < m < 2.5$. Note that this rate of convergence is optimal for W_2^1, despite the additional difficulties created by the numerical treatment of the 'Venttsel' condition.

Numerical Study for Rayleigh-Benard Convection in a Rectangular Box

V. Kolmychkov, O. Mazhorova, Yu. Popov and O. Shcheritsa

The report presents the numerical simulation of convective instability in a liquid with linear vertical initial temperature profile. Liquid is considered in rectangular box subjected to the gravity field. The simulation has been carried in 2D and 3D time-dependent approach. Fluid motion is described by the continuity equation, the Navier-Stokes equation in the Boussinesq-approximation and the heat transfer equation. The horizontal boundaries of the rectangular box are assumed to be rigid, vertical boundaries are heat-insulated.

The report provides results of stability analysis of the two-dimensional rolls with respect to 2D and 3D disturbances. Rayleigh number varies in range $1700 \div 10200$ and Prandtl number $Pr = 1, 0.71$. Computer simulations allow to determine stability boundaries. 2D numerical simulations shed light on the onset of convection. Different scenarios for onset of stable flow were registered. Comparison with the results of R. Krishnamurti, F.H. Busse and coworkers shows good agreement. In particular, Eckhause instability was observed.

Perturbation Bounds for Matrix Products and Numerical Solution of Matrix Equations

Mihail Konstantinov

Three main factors determine the accuracy of the numerical solution of matrix problems: the parameters of the finite arithmetic, the sensitivity of the computational problem and the properties of the numerical algorithm. The sensitivity and in particular the conditioning of the computational problems may be revealed by the techniques of perturbation analysis. Moreover, the inclusion of sensitivity and accuracy estimates is an important feature of modern computer codes for the solution of numerical linear algebra problems as well as the solution of algebraic and transcendental matrix equations.

In this paper we derive improved perturbation bounds for matrix products $Y = F(A) = A_1 A_2^{-1} A_3$, $A = (A_1, A_2, A_3)$, subject to perturbations $A_i \to A_i + \delta A_i$, where A_1, A_3 are arbitrary rectangular complex matrices and the matrix A_2 is square and invertible. The perturbation $\delta Y = F(A + \delta A) - F(A)$ in the matrix product Y is represented as $\delta Y = F_1(A, \delta A) + F_2(A, \delta A)$, where $\|F_k(A, E)\| = \mathrm{O}(\|\mathrm{E}\|^{\mathrm{k}})$, $E \to 0$, $k = 1, 2$. These results are applied to the local and non–local perturbation analysis of matrix equations $X = M + N^* X^p N$ and $X = M + N^* X(I + SX)^{-1} N$ arising in the theory of optimal control and filtering, where N^* is the complex conjugate transpose of the matrix N. Finally, the perturbation bounds obtained are implemented in condition and accuracy estimators in the computer codes for the solution of the corresponding matrix equations.

A Functional A Posteriori Error Estimate for Convection-Reaction-Diffusion Problems

Sergey Korotov, Dmitri Kuzmin and Antti Hannukainen

A new technique for global a posteriori error estimation for the stationary convection-reaction-diffusion problems is proposed. In order to estimate the approximation error in the usual energy norm, the underlying bilinear form is decomposed into a computable integral and two other terms which can be estimated from above using elementary tools of functional analysis. Two auxiliary parameter-functions are introduced to construct such a splitting and tune the resulting bound. If these functions are chosen in an optimal way, the exact energy norm of the error is recovered, which proves that the estimate is sharp. The presented methodology is completely independent of the numerical technique used to compute the approximate solution. In particular, it is applicable to approximations which fail to satisfy the Galerkin orthogonality, e.g. due to an inconsistent stabilization, flux limiting, low-order quadrature rules, round-off and iteration errors etc. Moreover, the only constant that appears in the proposed error estimate is global and stems from the Friedrichs-Poincaré inequality. Numerical experiments illustrate the potential of the proposed error estimation technique.

Robust Algebraic Multilevel Methods and Algorithms

Johannes Kraus, Svetozar Margenov

Preconditioners based on various multilevel extensions of two-level finite element methods (FEMs) lead to iterative methods which have an optimal order computational complexity with respect to the size (or discretization parameter) of the system. The methods can be on block matrix factorized form, recursively extended via certain matrix polynomial approximations of the arising Schur complement matrices or on additive, i.e. block diagonal form using stabilizations of the condition number at certain levels. The resulting spectral equivalence holds uniformly with respect to jumps in the coefficients of the differential operator. Following the spirit of the (A)lgebraic (M)ulti(L)evel (I)teration (AMLI) preconditioners we focus our attention to the robustness. This includes the case of strong mesh and/or coefficient anisotropy as well as parameter dependent ill-conditioned problems like almost incompressible elasticity and large Reynolds number in Navier-Stokes equations. Only two parameters, the constant in the strengthened Cauchy-Bunyakowski-Schwarz inequality (CBS constant) and the relative condition number of the preconditioner for the first pivot block in the recursive two-level splitting, determine the optimality of the methods and their robustness. The coefficient jumps are shaped automatically, while as we shall see, the robustness with respect to anisotropy (or other parameter dependent ill-conditioning) needs some special constructions.

The first part of the talk is focused on the AMLI theory for strongly anisotropic FEM elliptic systems. The cases of coefficient and mesh anisotropy for conforming (Courant) and nonconforming (Crouzeix-Raviart) linear finite elements are considered. The second part is addressed to some more recent results in AMLI methods including certain principle generalizations applied to the case of discontinuous Galerkin approximations. Selected numerical results are included to illustrate the potential of the optimal complexity AMLI methods for iterative solution of large-scale elliptic systems.

Optimal Numerical Parametrization of Curves and Surfaces

Evgenii Kuznetsov

Numerous mathematical models may be described by the system of nonlinear algebraic or transcendental equations

$$F_i(x_1, \dots, x_n, p_1, \dots, p_m) = 0, \quad i = 1, 2, \dots, n, \tag{1}$$

containing continuously varying parameters $p_1, p_2, \dots, p_m$ belonging to some set $P : \{p_l^0 \leq p_l \leq p_l^*,\ l = 1, 2, \dots, m\}$.

Let a solution of system (1) for some values of parameters be known, i.e.,

$$x_i = x_{i0} \quad \text{for} \quad p_l = p_{l0}, \quad i = 1, 2, \dots, n, \quad l = 1, 2, \dots, m.$$

Then the solution of system (1) for other values of parameters in the set P can be obtained using the method of solution continuation with respect to the parameters.

Definition *The subspace formed by vectors orthogonal to the rows of the Jacobi matrix of system (1) is referred as the tangent subspace.*

The following statement is valid.

Theorem *A system of linear equations for the continuation solution is the best conditioned if and only if, at any point of the smooth subspace of solutions set of the system of nonlinear equations (1), continuation parameters are chosen as these parameters the lengths of arches calculated along m vectors of orthonormal basis, laying into the tangent subspace.*

It is shown that the optimal parameter is the length of the curve to be interpolated. If the length of the polyline is used as a parameter, the parametrization is close to the optimal one; moreover, this parameter is optimal for the parametric approximation problem. For the parametric approximation of a surface, the optimal parametrization at each surface point is given by two orthogonal curves lying on the surface and passing through this point. The optimal parameters are the lengths of the arcs of those curves.

Approximate Subgradient Projection Algorithm for the Convex Feasibility Problem

Li Li, Yan Gao

The convex feasibility problem is to find a point $x^* \in R^n$ that satisfies the inequalities $f_i(x^*) \leq 0, i = 1, \ldots, m$, where each $f_i(x)$ is convex function defined on R^n. A ε-subgradient projection algorithm is presented for solving the convex feasibility problem. Based on the surrogate projection methods, a series of special projection hyperplanes are established. Moreover, compared with the pre-existing projection hyperplanes with subgradient projection algorithms, those special hyperplanes proposed in the paper are interactive with ε and its ranges are more larger. The convergence of the proposed algorithm is given under some mild conditions. Numerical tests are listed and the results generated are really impressive, which indicate the ε-subgradient projection algorithm for solving the convex feasibility problem is promising.

Comparative Analysis of High Performance Solvers for 3D Elasticity Problems

Ivan Lirkov, Yavor Vutov, Maria Ganzha and Marcin Paprzycki

In our work we consider numerical solution of 3D linear elasticity equations. The investigated problem is described by a coupled system of second order elliptic partial differential equations. This system is then discretized by conforming or nonconforming finite elements. After applying the Finite Element Method (FEM) based discretization, a system of linear algebraic equations has to be solved. In this system the stiffness matrix is large, sparse and symmetric positive definite. In our work we utilize a well-known fact that the preconditioned conjugate gradient method is the best tool for efficient solution of large-scale symmetric systems with sparse positive definite matrices. In this context, the displacement decomposition (DD) technique is applied at the first step to construct a preconditioner that is based on a decoupled block diagonal part of the original matrix. Then two preconditioners, namely the Modified Incomplete Cholesky factorization MIC(0) and the Circulant Block-Factorization (CBF) preconditioning, are used to precondition thus obtained block diagonal matrix.

As far as the implementation of the proposed solution methods is concerned, we utilize the Message

Passing Interface (MPI) communication libraries. The aim of our work is to compare the performance of the two proposed preconditioners: DD MIC(0) and DD CBF. The presented comparative analysis is based on the execution times of actual codes run on modern parallel computers. The performed numerical tests on parallel computer systems demonstrate the level of parallel efficiency and robustness of the proposed algorithms. Furthermore, we discuss the number of iterations resulting from utilization of both preconditioners.

Accuracy estimates of finite-difference method for Poisson equation, taking into account boundary effect

V. Makarov, L. Demkiv

Poisson equation in polyhedral domain $\Omega \subset \mathrm{R}^n$, $n = 2, 3$ with boundary Γ, when Dirichlet conditions are given on all faces or on all but one where Neimann conditions are given, is considered.

Traditional difference schemes with semi-constant steps along the axes is constructed, but in such a way that mesh boundary γ belongs to Γ.

Such schemes precisely approximate Dirichlet conditions hence one should expect that their accuracy order increases approaching to corresponding part of boundary γ.

This paper is dedicated to quantitative investigation of this boundary effect.

As a tool one uses discrete maximum principle and new estimates of Green difference functions for canonical domains (square, cube).

It is shown that analogous boundary effect in the mesh knots takes place also for finite-element method (super convergence).

Numerical Approaches to Scientific Information Search Systems

Vasil Marinov

This paper presents a novel approach to constructing of a searching system and its numerical realization. The method proposes a convenient design in the coding that improves the possibility of high-speed receiving corroboration of the hypotheses, good deal of examples with asked list of properties, with corresponding information if necessary. The control part in the coding that corresponds to the telecommunications is linked with the information part that is organized in a simplified manner and that describes in sufficient details the notions, conceptions and constructions of a global information system. The purpose is to cover the propositions as well the case, when they are not formal-logically contradictory. Some proofs of the properties of the code are done with consequent estimation of the number of combination in the surjective and injective homomorphisms cases is made.

The powerfulness of the method and of its course of action is illustrated through dealing with scientific information. The numerical technique is shown by examples considering various mathematical information.

Damping Control Strategies for Vibration Isolation of Disturbed Structures

Daniela Marinova

Dealing with vibration isolation of disturbed structures the active strategies are less commonly used than passive ones due to their cost and power requirements. Semi-active approaches can deliver the higher performance of fully active structures with less power consumption. This paper presents a semi-active damping control strategies based on balance control for a structural vibration isolation problem.

Control constraints and evaluation in terms of root-mean-square acceleration transmissibility are presented for the design problem. Numerical investigations and simulations facilitate the comparisons with the passive damper structures and show the efficiency of the proposed semi-active control strategies.

Numerical Simulation of Non-stationary Flow on Block Regular Grids

S. Martyushov, Y. Martyushova

Use of structured grids in calculation of non stationary gas dynamics problems separately in sub regions has some advantages before use of unstructured grids.

During the parallel calculation in sub areas it appears the necessity of formulation the internal boundary conditions with accuracy of difference algorithm, interface of results of calculation in sub areas with various time steps and definition of the order of accuracy of algorithm in time direction on the internal boundaries.

As usual in the problems of flow the calculation grid connected with streamline body is used. In case of non-stationary flows (for example diffraction of a shock wave on the flying body) the impulse of force generates non stationary (forward and rotary) movement of the body. The including of non-stationary speed of the body (and of connected calculation grid) in the basic equations and boundary conditions is most simply realized in the finite volume method.

For numerical simulation the TVD -scheme of the second order of accuracy was used. Construction of calculation grid was made by algorithm, based on the numerical decision of systems of Poisson and Beltramy equations. Two problems of interaction of the body with the shock wave in the shock tube of square section have been calculated. The pictures of flow, hardly visualized in natural experiment, where received. Qualitative comparison of calculation results with the integral characteristics of flow, received in experiments was made. Comparison has shown applicability of the algorithm for the numerical decision of such class of problems. Numerical simulation of these flows has continued a cycle of numerical calculations of diffraction of a shock wave on a flying body.

On the Effective Method for Numerical Solving Volterra Integral Equation

G. Mehdiyeva, V. Ibrahimov

Let's consider following nonline Volterra integral equation of the second kind

$$y(x) = f(x) + \int_{x_0}^{x} F(x, s, y(s))ds, \quad x \in [x_0, X]. \tag{1}$$

Let's suppose that continuous function $F(x, s, y)$ is defined on $G = \{x_0 \le s,\ x \le X,\ |y| \le a\}$ and here has continuous partial derivative to some p order inclusively.

The first is considered as relation between methods quadrate and Adam's. By means of these relations of the quadraturial methods, lets add the next form

$$\sum_{i=0}^{k} \hat{\alpha}_i y_{n+i} = \sum_{i=0}^{k} \hat{\alpha}_i f_{n+i} + h \sum_{i=0}^{k} \gamma_i F_{n+i}, \tag{2}$$

k – is integral value dimension, $0 < h$ is constant step which divided the segment $[x_0, X]$ into N equal parts, α_i, β_i $(i = 0, 1, 2, \ldots, k)$ some real numbers, y_m approximately values of solution of equation (1) in point $x_m = x_0 + mh$ $(m = 0, 1, 2, \ldots)$ and

$$f_m = f(x_m), \quad F_m = F(x_m, x_m, y_m) \quad (m = 0, 1, 2, \ldots).$$

According to shortened methods (2) the following k-step methods with constant coefficients is suggested for numerical solution of equation (1)

$$\sum_{i=0}^{k} \alpha_i y_{n+i} = \sum_{i=0}^{k} \alpha_i f_{n+i} + h \sum_{j=0}^{k} \sum_{i=0}^{k} \beta_i^{(j)} K(x_{n+j}, x_{n+i}, y_{n+i}). \tag{3}$$

Note that the coefficients α_i and $\beta_i^{(j)}$ $(i, j = 0, 1, \ldots, k)$ can define by the coefficients $\hat{\alpha}_i$ and γ_i $(i = 0, 1, \ldots, k)$.

Numerical solution of a class of boundary value problems arising in the physics of Josephson junctions

Hristo Melemov, Todor Boyadjiev

A method for numerical solution of non-linear boundary value problems for systems of ODE's given on the embedded intervals, is described. Similar problems take place, for example, in physics of stacked Josephson junction with different layers lengths. The algorithm is based on the continuous analog of Newton method coupled with spline-collocation scheme of high order of accuracy.

Two cases of such junctions — geometrically symmetric and asymmetric are considered. The influence of length's ratio on the main physical properties of basic magnetic flux distributions is investigated numerically.

How to choose basis functions in some meshless methods

Vratislava Mošová

Technical progress in construction of computers, its higher speed and larger memory, gave possibility to develop new numerical methods for solution of problems that are described by means of differential equations. A lot of meshless methods have been developed by engineers in last years.

While classical numerical methods such as Finite Difference Method, Finite Element Method or Finite Volume Method need explicitly given mesh, the meshless methods such as EFG (Element free Galerkin), RKPM (Reproducing Kernel Partition of Unity), FEM with B-splines, GFEM (Generalized Finite Element Method) do not need any explicitly given mesh at the beginning of computation. Moreover, the classical numerical method uses local approximation by polynomials. The meshless methods construct new shape functions that do not have to be only polynomial.

Some meshless methods are in a way identical with the Galerkin method, where the trial space is formed by specially constructed functions. The choice of the proper trial space is important, because the error in the Galerkin method is determined by how well the exact solution can be approximated by the elements from this finite dimensional space.

It is the main reason why in this contribution different trial spaces are studied. We will consider spaces generated by means of B-splines and shape functions received by means of RKPM or MLS technique. We focus our attention on error estimates in these cases. Examples of using mentioned trial spaces for solution of some boundary value problems will be given in the end.

About Accuracy of k-Step Method with the Third Derivatives

Inara Nasirova

Consider the following Cauchy problem:

$$y''' = f(x, y, y', y''), \quad \left.\frac{d^j y(x)}{dx^j}\right|_{x=x_0} = y^{(j)}(x_0) \ \ (j = 1, 2), \ \ y(x_0) = y_0. \tag{1}$$

Let us assume that, the solution (1) has the unique solution defined on the segment $[x_0, X]$. By means of $0 < h$ the segment $[x_0, X]$-divided into N equal parts with $x_i = x_0 + ih$ $(i = 0, 1, 2, ...)$ points. Let us denote with y_i-approximate, and $y(x_i)$-exactly values of the solving problems (1) in a point x_i $(i = 0, 1, 2, ...)$. For definition of values y_i $(i = 0, 1, 2, ...)$ is suggested herein the following forward jumping method:

$$\sum_{i=0}^{k-m} \alpha_i y_{n+i} = h \sum_{i=0}^{k} \beta_i^{(1)} y'_{n+i} + h^2 \sum_{i=0}^{k} \beta_i^{(2)} y''_{n+i} + h^3 \sum_{i=0}^{k} \beta_i^{(3)} f_{n+i}. \tag{2}$$

Here $\alpha_\nu, \beta_i^{(j)}$ $(\nu = 0, 1, ..., k - m; j = 1, 2, 3; i = 0, 1, 2, ..., k)$ some of real numbers, at that $\alpha_{k-m} \neq 0$ and $\left|\beta_k^{(1)}\right| + \left|\beta_k^{(2)}\right| + \left|\beta_k^{(3)}\right| \neq 0$, k and m - are integer values, $m \geq 1$, and $f_\nu = f(x_\nu, y_\nu, y'_\nu, y''_\nu)$ $(\nu \geq 0)$. Application of method (2) is accompanied with some difficulties. For example, consider the following method

$$y_n = y_{n-1} + h(\beta_0^{(1)} y'_{n-1} + \beta_1^{(1)} y'_n + \beta_2^{(1)} y'_{n+1}) +$$

$$+ h^2(\beta_0^{(2)} y''_{n-1} + \beta_1^{(2)} y''_n + \beta_2^{(2)} y''_{n+1}) + h^3(\beta_0^{(3)} f_{n-1} + \beta_1^{(3)} f_n + \beta_2^{(3)} f_{n+1}). \tag{3}$$

While defining dimensions of the value y_n, dimensions of some of the values $y_{n-\nu}$ $(\nu = 1, 2, 3)$ are usually used. However, for determination of the approximate value of the solution of solving (1) y_n in the point x_n by the method (3), it is necessary to know values of the solution of solution (1) and in the next point x_{n+1}, that is usually determined by the values of y_n. For elimination of the indicated deficiency, a special scheme is offered, that can be referred to as the predictor-corrector method.

Question of Existence and Uniqueness of Solution for Navier–Stokes Equation for Linear "do-nothing" type Boundary Condition on the Outflow

Tomáš Neustupa

The paper is concerned with the theoretical analysis of the model of incompressible, viscous, stationary flow through a plane cascade of profiles. The boundary value problem for the Navier-Stokes system is formulated in a domain representing the exterior to an infinite row of profiles, periodically spaced in one direction. Then the problem is reformulated in a bounded domain of the form of one space period and completed by the Dirichlet boundary condition on the inlet and the profile, a suitable natural boundary condition on the outlet and periodic boundary conditions on artificial cuts. Specially, we study the question of uniqueness of the weak solution of this problem for linear separated "do nothing" type boundary condition (which we derive)

$$q = h_1, \qquad -\nu\,\omega(\boldsymbol{u}) = h_2$$

and for nonlinear modification of the "do nothing" type of boundary condition (proposed by C. H. Bruneau, F. Fabrie)

$$-\nu\,\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}} + p\boldsymbol{n} - \frac{1}{2}\,(\boldsymbol{u}\cdot\boldsymbol{n})^{-}\boldsymbol{u} \,=\, \boldsymbol{h}.$$

The problems of existence and uniqueness for these cases are discussed and compared.

Preconditioners for Block Matrix

Felicja Okulicka - Dłuzewska

Description of different mechanical phenomena such as flow, mechanical behavior, thermal effects, leads to coupled systems of differential equations. The finite element method is widely used to solve such problems. The most important part of the finite element method algorithm is the procedure of solving the set of linear and non-linear equations. In the case when the coupled problems are taken under consideration the structure of the system of equations leads to the application of the block solvers. Each block describes different phenomena or the coupled part, has different condition coefficient and can be calculated independently from others. The large systems of linear equation, especially in the situation when matrices are sparse or bounded, can be easily solved by the parallel iterative method. This method with block structure of the matrix can also be very easy implemented on distributed memory system. The application of the iterative methods unfortunately can lead to the problems with convergence. In the paper the iterative methods with different preconditioners applied to the diagonal blocks of matrix are tested and compared.

Geometrical analysis of Model Predictive Control. A parameterized polyhedra approach

S. Olaru, I. Dumitrache and D. Dumur

Model Predictive Control (MPC) is an effective technique for the control of systems with constraints. This optimization based control technique applies in a receding horizon manner the solution of an open loop optimal control problem. In the case of linear system this leads to linear/quadratic optimisation problems. Due to the fact that the procedure is reiterated at the next sampling time, the resulting optimisation problem is parameterized by the state vector of the system.

In the case of linear time-invariant systems the feasibility domains of the predictive control problems are represented as parameterized polyhedra. The double representation of these sets in terms of constraints and generators provide a valuable insight into the topology of the optimisation problems to be solved at each sampling time. Practically, the constraints redundancy can be described by expressing the validity domain of the parameterized vertices and further the explicit control law can be obtained by using the Abadie constraints qualification for optimality. In the case of non-unique optimal solution (for example in multiparametric linear programming), the entire family of solution can be obtained.

Finally, the construction of invariant sets can be used to provide feasibility and stability guarantees, completing this geometrical study of the model-based control strategies.

An interesting research direction is provided by the class of hybrid system, the model predictive control leading in this case to multiparametric mixed integer optimisation problems. In this framework, the parameterized polyhedra approach can be combined with the Voronoi diagrams to obtain analytic solutions and thus provide explicit control laws in terms of both continuous and switching control actions.

An iterative numerical algorithm for a coupled system of singularly perturbed convection diffusion two point boundary value problems

Eugene O' Riordan, Jeanne Stynes and Martin Stynes

Consider the following system of $m \geq 2$ singularly perturbed convection-diffusion-reaction ordinary differential equations

$$L\boldsymbol{u} := (-E\boldsymbol{u}'' - B\boldsymbol{u}' + A\boldsymbol{u})(x) = \boldsymbol{f}(x), \quad x \in (0,1) \tag{1}$$

in the unknown vector function $\boldsymbol{u}$, where the boundary values $\boldsymbol{u}(0)$ and $\boldsymbol{u}(1)$ are given. The $m \times m$ matrices $A = (a_{ij})$ and $B = (b_{ij})$ have entries in $C^3[0,1]$ and $E = diag\{(\varepsilon_1, \varepsilon_2 \dots \varepsilon_m)\}$ is a constant diagonal matrix with $0 < \varepsilon_i \leq 1$.

If $B \equiv 0$ then the above system is said to be of reaction–diffusion type. Numerous recent publications have studied reaction-diffusion systems and have analysed parameter–uniform numerical methods for these problems using suitably fitted meshes. Most of these papers impose conditions on the matrix A, such as an M-matrix structure, so that the resulting operator is of inverse-monotone type. If B is a diagonal non–zero matrix, then the system (1) is said to be weakly coupled. If

B has a non-zero entry off the main diagonal then we say that (1) is *strongly coupled*. Although the analysis in some publications does not rely on inverse-monotonicity of the differential operator, it is not obvious how to extend these analytical techniques to the case of a strongly coupled convection-diffusion system. Note in particular that a strongly coupled system will not in general satisfy a maximum principle.

In contrast to the reaction-diffusion case, there have been relatively few publications on systems involving strongly coupled convection-diffusion equations. In [5], a first-order parameter-uniform method is analysed, under the assumptions that $m = 2$ and that B is a strictly diagonally dominant M-matrix. It is difficult to see how to generalize this analysis to the case of $m > 2$. The papers [3, 4] employ scalar stability inequalities from Andreev [1] to derive parameter-uniform error bounds. This analysis requires B to be strongly diagonally dominant. Our analysis in the present paper resembles that of [4], but here we employ an iterative Jacobi algorithm (cf. [2]) to analyse the continuous solution and to generate the numerical solution. In this paper, we assume that

$$\min_{1\le i\le m}\min_{x\in[0,1]} b_{ii}(x) > 0, \qquad \min_{1\le i\le m}\min_{x\in[0,1]} a_{ii}(x) \ge 0$$

$$\max_{1\le i\le m}\sum_{\substack{j=1\\ j\neq i}}^{m}\left(\beta_i^{-1}\|b'_{ij}+a_{ij}\|_{L_1}+R_i\|b_{ij}\|_\infty\right) < 1,$$

$$\text{where}\qquad R_i := \left(1+\frac{\|a_{ii}\|_\infty}{\beta_i}\right)\left(\int_{x=0}^{1}\left|\left(\frac{1}{b_{ii}(x)}\right)'\right|\,dx+\frac{2}{\beta_i}\right).$$

We construct an iterative numerical method for the system (1), which combines a standard finite difference operator with a piecewise-uniform Shishkin mesh. Under the above assumptions on the coefficients and the data, we establish a parameter-uniform asymptotic error bound and present some numerical results to illustrate the performance of the iterative method.

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Parallel Performance and Scalability Experiments with the Danish Eulerian Model on the EPCC Supercomputers

Tzvetan Ostromsky, Ivan Dimov and Zahari Zlatev

The Danish Eulerian Model (DEM) is a powerful air pollution model, designed to calculate the concentrations of various dangerous species over a large geographical region (e.g. Europe). It takes into account the main physical and chemical processes between these species, the actual meteorological conditions, emissions, etc. . This is a huge computational task and requires significant resources of storage and CPU time. Parallel computing is essential for the efficient practical use of the model. Some efficient parallel versions of the model were created over the past several years.

A suitable parallel MPI version of DEM was implemented on two powerful supercomputers of the EPCC - Edinburgh, available via the HPC-Europa programme for transnational access to research infrastructures in EC: a Sun Fire E15K and an IBM HPCx cluster. Although the implementation is in principal, the same for both supercomputers, few modifications had to be done for successful porting of the code on the IBM HPCx cluster. Performance analysis and parallel optimization was done next. Results from benchmarking experiments will be presented in this paper.

Another set of experiments was carried out in order to investigate the sensitivity of the model to variation of some chemical rate constants in the chemical submodel.Certain modifications of the code was necessary to be done in accordance with this task. The obtained results will be used for further sensitivity analysis studies by using Monte Carlo simulation.

LAPACK-based Condition Estimates for the Discrete-time LQG Design

P. Petkov, M. Konstantinov and N. Christov

The Linear-Quadratic Gaussian (LQG) design is the most efficient and widely used design approach in the field of linear stochastic control systems. From theoretical point of view this approach is reduced to the synthesis of a LQ state regulator and of a Kalman filter for the controlled system. From computational point of view the LQG design consists of solving a pair of matrix Riccati equations: one for the LQ regulator design and a second one (dual to the first Riccati equation) for the Kalman filter design.

In this paper we present reliable algorithms for estimation of condition numbers of the discrete Riccati equations in the discrete-time LQG design. The basic calculation for obtaining these estimates is the evaluation of the 1-norm of the inverse of a Lyapunov operator. This is performed by evaluating this operator and its dual on suitably selected values, or, equivalently, by solving related Lyapunov equations. Various strategies for exploiting the symmetric structure of the problem are investigated and compared. Efficient, LAPACK-based condition estimators are proposed involving the solution of discrete Lyapunov equations.

Analysis of Expected Climate Change in the Carpathian Basin Using a Dynamical climate Model

I. Pieczka, J. Bartholy, R. Pongrácz and A. Hunyady,

Global climate models (GCM) are widely used for estimating the possible global warming due to increased anthropogenic influences. The results from these coarse resolution (about 300 km) models can only be considered as a first-guess of regional climate change consequences of the global warming. Regional climate models (RCM) are dynamical models nested in GCMs and they may lead to a much better estimation of future climate conditions in the European subregions since the horizontal resolution of these RCMs is much finer (maybe as fine as 10-25 km) than the GCMs. Moreover, high resolution model results are essential for the generation of national climate change scenarios, as it is recommended by the United Nations Development Programme (UNDP). For analyzing the possible regional climate change in the Carpathian Basin, we have adapted the model PRECIS at the Department of Meteorology, Eötvös Loránd University. The model PRECIS is a hydrostatic regional climate model HadRM3P developed at the UK Met Office, Hadley Centre, and nested in HadCM3 GCM. It uses 25 km horizontal resolution transposed to the Equator and 19 vertical levels with sigma coordinates. First, we evaluate the model capability of reconstructing the present climate (1961-1990) using two different sets of boundary conditions, (i) from the European Centre for Medium Range Weather Forecast ERA-40 reanalysis database, (ii) from the HadAM3P GCM output data. In order to fulfill the validation task the results of the different model experiments are compared to the monthly climatological datasets of the Climatic Research Unit (CRU) of the University of East Anglia as a reference. Then, we compare the model results for the periods 2071-2100 (using SRES A2 scenario) and 1961-1990 (as the reference period).

Finite-Difference Method for Computation of the Gas Dynamics Equations with Artificial Viscosity

Igor Popov, Igor Fryazinov

Finite-difference method for computation of the gas dynamics equations with artificial viscosity is proposed. It is homogeneous, monotonous finite-difference scheme of the second order approximation on time and space variables outside of areas of breaks and compression waves. The paper presents new way of introduction of artificial viscosity. It is investigated stability of the scheme. Test calculations of contact breaks movement, shock waves and disintegration of breaks were performed.

Using Additive Splitting for Numerical Solving the Multidimensional Convection-Diffusion Problem

V. Prusov, A. Doroshenko and R. Chernysh

An efficient method for numerical solving the one-dimensional convection-diffusion problem was constructed before. It has such properties of explicit and implicit schemes which present us a practical interest. The method is economical to run as well as an explicit scheme since recurrent formulas are used for computation. The method is absolutely stable on acceptable grids as well as an implicit scheme.

A splitting based on directional decomposition to solve multidimensional convection-diffusion problem is considered. It is additive and suitable for the method of numerical solution of the one- dimensional problem. The splitting is based on independent solving several one-dimensional subproblems with the same initial value in each directions and summarizing the results at each time step of the numerical integration.

The three-dimensional convection-diffusion problem which has an analytical solution is solved by using the splitting and the efficient method. For comparison it is also solved by using explicit and implicit upstream schemes. All numerical results are compared with the analytical solution.

The Fourier Spectral Method for the Sivashinsky Equation

Abdur Rashid, Ahmad Izani Md. Ismail

In this paper, a Fourier spectral method for solving the Sivashinsky equation with periodic boundary conditions is developed. We establish semi-discrete and fully discrete schemes of the Fourier spectral method. A fully discrete schemes is constructed in such a way that the linear part is treated implicitly and the nonlinear part explicitly. We use an energy estimation method to obtain error estimates for the approximate solutions. We perform some numerical results.

The Grid Method for Solving the Traffic Flow Problem on the Highway in a Class of Discontinuous Functions

Mahir Rasulov, Kenan Gocer

This study suggests a new method for obtaining an exact and a numerical solution of the initial value problem for a first- order one dimensional nonlinear partial equation which describes the macroscopic traffic flow on highways in a class of discontinuous functions.

Using the first approach, in the traffic flow problem the flux function $q(x,t)$ is expressed by the local density $\rho(x,t)$ as $q = Q(\rho) = \rho V(\rho)$. If we assume that there will be no vehicle entering or leaving the highway, that the functions $\rho(x,t)$ and $Q(\rho)$ are continuously differentiable the equation which describes the traffic flow is

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial Q(\rho(x,t))}{\partial x} = 0.$$

From a physical point of view the function $Q(\rho)$ must be concave, that is $Q^{''}(\rho) \leq 0$, and the initial distribution of vehicles on the highway may be a continuous function as well as piecewise.

It is known that the solution of this problem has the points of discontinuity of which the location is not known beforehand and that this solution also satisfies the entropy condition.

The special auxiliary problem having some advantages over the main problem is suggested. Using the solution of the auxiliary problem an original method for finding the weak solution of the main problem is proposed. The solution obtained on the basis of the auxiliary problem expresses all physical properties accurately.

The Numerical Study of a Two-Phase Fluid in a Porous Medium in a Class of Discontinuous Functions

Mahir Rasulov, Haluk Kul

It is known that the motion of the multi-phase fluid through a porous medium is modelled by the system of mixed nonlinear partial differential equations called as Muckat-Mires equations. Appropriate conditions are chosen in accordance with the strategies of exploitation of oil reservoirs.

In this study, the Muckat-Mires's system of equations is decomposed with respect to physical factors to two equations as

$$div\left[k\left(\frac{k_o(\sigma)}{\mu_o}+\frac{k_w(\sigma)}{\mu_w}\right)gradp\right]=-q \tag{1}$$

$$m\frac{\partial\sigma}{\partial t}+div\left[w(t)F_w(\sigma)\right]=q_w, \tag{2}$$

The system of equations is studied under the following initial and boundary conditions

$$\frac{\partial p}{\partial\vec{n}}\Big|_{\partial D}=0, \sigma_w(x,y,0)=\sigma_w^{(0)}, \quad \sigma_w\Big|_{\Gamma_\nu^{(w)}}=1-\sigma_w^{(0)}. \tag{3}$$

Here $\sigma_w^{(0)}$ is the value of connected water, $p_0(x,y)$-initial distribution of pressure of oil phase; $\Gamma_\nu^{(w)}=\left\{\left(x_\nu^{(w)},y_\nu^{(w)}\right), \nu=1,2,...,N_1\right\}$ are coordinates of the water wells.

The efficient numerical method for solving the problem (1)-(3) is developed. The obtained solution is sensitive, and it accurately describes all physical properties and permits the evaluation of the entire technological significance of the exploitation.

Investigated the Quality of Water on The Model of Shallow Water Flow Over an Isolated Ringe in a Class of Discontinuous Functions

Mahir Rasulov, Veysel Kilic and Rifat Colkesen

A finite difference method for solving the initial boundary value problem for the one dimensional nonlinear system equations for investigated the quality of water on the model of Shallow Water flow over on isolated ringe in a class of discontinuous functions is suggested. In order to develop the numerical algorithm the special auxiliary problem having some advantages over the main problem is introduced. The solution obtained from the auxiliary problem represent the all physical nature of the investigated problem with a high accuracy.

A Hybrid Genetic Algorithm for Parameter Identification of Bioprocess Models

Olympia Roeva

Modeling approaches are central in system biology and provide new ways towards the analysis and understanding of cells and organisms. A common approach to model cellular dynamics is the sets of nonlinear differential equations. Parameter identification of the nonlinear dynamic model is more difficult than that of the linear one, as no general analytic results exist. The difficulties that may arise are such as convergence to local solutions if standard local methods are used, over-determined models, badly scaled model function, etc. Due to the nonlinearity and constrained nature of the considered systems, these problems are very often multimodal. Thus, traditional gradient-based methods may fail to identify the global solution and there is a distinct need for using global optimization methods which provide more guarantees of converging to the globally optimal solution. Although a lot of different global optimization methods exist, the efficacy of an optimization method is always problem-specific. In this paper, the Genetic algorithms (GA), are examined.

The principal attractions of GA are domain independence, non-linearity and robustness. The only requirement for GA is the ability to calculate the measure of performance which may be highly complicated and non-linear. The above two characteristics of GA assume that GA is inherently robust. They can work with highly non-linear functions and can cope with a great diversity of problems from different fields.

However, the conventional GA has a very poor local performance because of the random search used. To achieve a good solution, great computational cost is inevitable. The same qualities that make the GA so robust also can make it more computationally intensive and slower than other methods.

To improve the performance of the conventional GA, a hybrid scheme using the GA and sequential quadratic programming (SQP) method is introduced. The hybrid optimization approach presented in this work switches between global and local search methods. Thus, optimizers work jointly to efficiently locate quality design points better that either could alone. GA can reach the region near an optimum point relatively quickly, but it can take many function evaluations to achieve convergence. The technique used here is to run GA for a small number of generations to get near an optimum point. Then the solution from GA is used as an initial point for the SQP method that is faster and more efficient for local search. To demonstrate the usefulness of the approach presented, two case studies of differing complexity are considered. The hybrid scheme is applied for parameter identification of two systems of nonlinear differential equations modeling the fed-batch cultivation process of the bacteria *E. coli.* The results show that using the hybrid function can greatly reduce the computational cost for achieving a desired accuracy.

Approximation of Singularly Perturbed Parabolic Equations in Unbounded Domains in the Case of Solutions Growing at Infinity

Grigory Shishkin

An initial–boundary value problem is considered in an unbounded in x domain (on a semiaxis) for a singularly perturbed parabolic reaction–diffusion equaiton; the highest order derivative is multiplied by a parameter ε^2, $\varepsilon \in (0, 1]$. The right-hand side of the equation and the initial function for $x \to \infty$ increase unboundedly as $O(\Psi(x))$, where $\Psi(x) = 1 + x^2$, which leads to the unbounded growth of the solution at infinity. The initial–boundary value function is pieciewise uniform. For small values of the parameter ε, boundary and an interior layers arises, respectively, in a neighbourhood of the lateral part of the boundary and in a neighbourhood of the characteristics of the reduced equation passing through the points of nonsmoothness of the initial function. In this problem, the error of numerical solutions grows without bound as $x \to \infty$ even for fixed values of the parameter ε.

In the present paper, the proximity of solutions of the initial–boundary value problem and of its numerical approximations is considered in a weighted maximum norm $||\cdot||^w$ with the weight function $\Psi^{-1}(x)$; in this norm the solution of initial–boundary value problem is ε-uniformly bounded. Using the method of special grids condensing in neighbourhoods of the boundary and interior layers, we construct and study special finite difference schemes that converge ε-uniformly in the weighted norm. It is shown that the convergence rate of the schemes essentially depends on the type of nonsmoothness in the initial–boundary conditions. Also, numerical approximations converging ε-uniformly in the weighted norm are considered for the Cauchy problem whose right-hand side and initial function grow as $O(\Psi(x))$.

Higher-Order Accurate Numerical Methods for Singularly Perturbed Parabolic Equations with Discontinuous Initial–Boundary Conditions

Grigory Shishkin

A Dirichlet problem is considered for a singularly perturbed parabolic reaction–diffusion equation in a rectangular domain; the initial-boundary conditions are piecewise smooth. The highest order derivative is multiplied by a parameter ε^2; $\varepsilon \in (0, 1]$. For fixed values of the parameter ε, the solution of the problem has singularities such as a discontinuity of the solution. For small values of ε, a boundary and an interior layer (with the typical width ε) arises, respectively, in a neighbourhood of the lateral part of the boundary and in a neighbourhood of the characteristic of the reduced equation passing through the point of discontinuity of the initial function.

Special finite difference schemes that converge ε-uniformly are constructed on the basis of the method of splitting of singularities generated by discontinuities of the boundary function and its lowest derivatives (low-order derivatives). The convergence rate of the schemes is improved using the methods of increasing the accuracy developed for boundary value problems with smooth solutions.

Grid Approximation of Singularly Perturbed Parabolic Equations on a Ball in the Case of a Third–Kind Boundary Condition

Lidia Shishkina, Grigory Shishkin

On a ball, we consider a boundary value problem for a singularly perturbed parabolic reaction–diffusion equaiton; the third–kind boundary condition admitting both Dirichlet and Neumann conditions is given on the boundary of the domain. The perturbation parameter ε multiplying the space derivatives takes arbitrary values in the half-open interval $(0, 1]$. For such a problem, solutions of standard difference schemes and also diffusion fluxes obtained using the discrete solutions even qualitatively do not approximate the real solutions and flows.

Using the method of condensing grids, we construct a finite difference scheme whose solution, as well as the resulting corresponding diffusion fluxes, converges ε-uniformly. Problems of this type arise, for example, in mathematical modelling of drying processes for dispersion materials in tube driers.

Properties And Applications Of Generalized Polynomial Spaces In Three Variable

Dana Simian

Multivariate interpolation is a topic which often appears in practical modeling problems. Different type of spaces of functions are used for solving interpolation problems. When the interpolation conditions are of different kind, by example, spacial and temporal, one possibility for modeling the problem is to use a generalize degree, in which the monomials exponents are weighted with a weight vector with integer components. In order to use such a generalize polynomial space as interpolation space, it is necessary to know the dimension and a basis of it. The aim of this article is to study and prove many properties of the generalize polynomial spaces in three variables and to use these spaces in a particular interpolation problem. Computational aspects are also discussed.

An Optimization Model For A Network Having Broken Packages

Dana Simian, Vladislav Georgiev and Corina Simian

Broken packages have a negative impact on a computer network. They affect the routers, the system load average is getting too big and the operating system crushes. When the broken packages arrive at the third level of the network, the router doesn't know what to do with them, because they don't have enough information. Due to this, the packages stay in the router too much time. This is a real problem which appears in the computer network of Sofia University. We need to find where these packages come from. The only information we have is the time when a broken package arrives in the router. The aim of this article is to built an mathematical model, temporal dependent and weights based, for optimizing the localization of the computers where the broken packages come from.

Applications of the connection between approximation theory and algebra

Dana Simian, Corina Simian

The aim of this paper is to illustrate a possibility of obtaining various theoretical results using the connection between multivariate interpolation and reduction process with respect of a H-basis of an ideal. Using this connection we can solve interpolation problems using only methods from algebra. As a application of this connection, we found an interesting differential equation, satisfied by the polynomials from a particular interpolation subspace. Choosing as interpolation conditions N points situated on the unit circle on R^d, if $N = \binom{2m+d}{d,}$, $m \in Z_+$, then, all polynomials u, from a minimal interpolation space with respect to these interpolation conditions satisfy the differential equation

$$\sum_{k_1=0}^{m} \sum_{k_2=0}^{k_1} \cdots \sum_{k_{d-1}=0}^{k_{d-2}} C_m^{k_1} C_{k_1}^{k_2} \dots C_{k_{d-2}}^{k_{d-1}} \cdot D^{(2(m-k_1),2(k_1-k_2),\dots,2(k_{d-2}-k_{d-1}),2k_{d-1})}(u) = 0$$

Particular examples, for univariate and bivariate case are given and analyzed.

Efficient Numerical Method of a One-Dimensional Motion of a Two-Phase Fluid Through a Porous Medium in a Class of Discontinuous Functions

Bahaddin Sinsoysal, Mahir Rasulov

This note is devoted to the study of a system of nonlinear partial differential equations

$$\frac{m}{k}\frac{\partial}{\partial t}\left(\frac{\sigma_o}{\beta_o}\right) = div\left(\frac{k_o}{\mu_o\beta_o} gradp_o\right) + q_o, \tag{1}$$

$$\frac{m}{k}\frac{\partial}{\partial t}\left(\frac{\sigma_w}{\beta_w}\right) = div\left(\frac{k_w}{\mu_w\beta_w} gradp_w\right) + q_w, \tag{2}$$

which describes the flow of a compressible two-phased mixture through a porous medium taking the capillary pressure into account.

The equations (1),(2) have been studied by Buckley-Leverett for the case $p_c(\sigma_w) = 0$. When $p_c(\sigma_w) \neq 0$, the idea was developed if the fluids are incompressible.

In this study, the system equations (1),(2) is split with respect to physical factors. The new method for finding the numerical solution of this system is suggested, if $p_c(\sigma_w) \neq 0$ and the fluids are compressible.

Travelling waves of the form of compactons, peakons, cuspons and their CNN realization

A. Slavova, P.Popivanov

This paper deals with the construction of travelling waves of some equations of mathematical physics as generalized Camassa-Holm equation, generalized KdV equation and several others. We construct compactly supported solutions as well as solutions forming singularities of the type peak (angle) and cusp. To do this tools of classical analysis and ODE are used. The CNN realization enables us to obtain numerical results and computer visualization of the corresponding solutions.

The Neural Networks Approach to Identification of Local Damages in Elastic Structures

A. Soloviev, P. Kourbatova, N. Saprounov

The Neural Networks Approach to Identification of Local Damages in Elastic Structures One application of artificial neural networks (ANN) is the resolving of coefficient geometric problems of elasticity applying to important practical area nondestructive evaluation (NDE) and defectoscopy. An efficiency of ANN in problems of determination and classification of different defects in structural materials is convincingly presented in the number of works that refer to using the ANN with applications to ultrasound defectoscopy. ANN technology was implemented in means of program supporting some equipment for defectoscopy and damages diagnostics. All NDE methods it is possible conditionally to divide onto two groups. First group include the methods utilized the monitoring of non mechanical values, such as vortex electric current, a magnetic flux, X-ray imaging, etc. The second group uses the measurements of mechanical quantities: an elastic wave speed, a frequency response and time dependent response of displacements or accelerations of points located on free sections of body surfaces accessible to measuring. The information necessary for defects identification usually derive in the discrete form. This is a spectrum of eigenfrequencies, a density of natural vibrations modes energy (a spectral probe), or oscillations amplitudes of finite set points on the body surface (a positional probe). For identification of damaged state, for reconstruction of defects kind and geometry the multilayered perceptron (MLP) architecture more often is used. Such MLPs were learned by error back propagation (BP) method. Creation of learning vectors require a considerable set of experimental data obtained at different condition or the other way it is require an efficient numerical solving method of forward dynamic problems for structures with previously well-known imperfections. For simplest structures, e.g. homogenous beams, plates, etc. it is possible to utilize the analytical representations. However, for practice applications the main means is finite element method (FEM). But some unconformity between FEM model behavior and measured vibration data can cause to mistaken or not responsible conclusions. These considerations have fairly indicated by some authors proposed the statistical approach for elimination of this deficiency. In the presented work a resolving method of mentioned problems is proposed. This method based on a linked FEM and ANN allows determining the geometrical properties of the boundary flaw, of cavities and holes in the structural elements. We suppose that the flaw-sides not interact and that inners boundaries of cavities are stresses - free. Thus the reconstruction of flaws was performed on the basis of spectral probing, and definition of the holes geometry (coordinates of center and reference size) - on the basis of positional probing. A learning of the developed networks was performed by finite-element software ACELAN working in batch mode and interacting with Neural Network Toolbox MATLAB 7.1 trough xml interface. The examples of feedforward network training by backpropagation algorithm and also examples of damages like surface flaws and holes were considered. The numerical experiment results were shown a good reliability of developed method and performance of its program implementation. The problem of sensors and actuators optimal placement on each structure with own structural anisotropy and defects kind is a direction of next studies.

Numerical Approximation of a Free Boundary Problem for a Predator-Prey Model

Răzvan Ştefănescu, Gabriel Dimitriu

The free boundary problems associated with the predator-prey ecological models have attracted considerable research attention due to their relevance in applications.

In this paper we consider a two-species predator-prey ecological model for $P(x,t)$ and $Q(x,t)$ with a free boundary $x = h(t)$, such that

$$\begin{cases} P_t - d_1 P_{xx} = P(a_1 - b_{11}P + c_{12}Q), & 0 < x < h(t),\ t > 0, \\ Q_t - d_2 Q_{xx} = Q(a_2 - b_{21}P - c_{22}Q), & 0 < x < l,\ t > 0, \\ P(x,t) \equiv 0, & h(t) < x < l,\ t > 0, \\ P = 0,\ h'(t) = -\mu \frac{\partial P}{\partial x}, & x = h(t),\ t > 0, \\ \frac{\partial P}{\partial x}(0,t) = \frac{\partial Q}{\partial x}(0,t) = \frac{\partial Q}{\partial x}(l,t) = 0, & t > 0, \\ h(0) = b,\ (0 < b < l), & \\ P(x,0) = P_0(x) \geq 0, & 0 \leq x \leq b, \\ Q(x,0) = Q_0(x) \geq 0, & 0 \leq x \leq l, \end{cases} \tag{1}$$

where d_i, a_i, b_{ij}, c_{ij} are positive constants. In biological terms, P and Q represent, respectively, the spatial densities of predator and prey species that are interacting and migrating in the habitat $(0,l)$, d_i denotes its respective diffusion rate and the real number a_i describes its net birth rate. b_{11} and c_{22} are the coefficients of intra-specific competitions, and b_{21} and c_{12} are the coefficients of inter-specific competitions. Also, we mention that the initial values P_0, Q_0 are nonnegative and satisfy $P_0(x) \in C^2[0,b]$, $P_0(x) > 0$ in $[0,b)$, $P_0'(b) < 0$, $Q_0(x) \in C^2[0,l]$, and the consistency conditions $P_0'(0) = Q'(0) = Q_0'(l) = 0$. Here, the positive constant μ on the interface (free boundary) depends on the d_1, the diffusivity of the predator. Taking into account the local dynamics of the system, a stable finite-difference scheme for the solution of (1) is applied and numerical results are presented.

On the time-step of the theta-method in the discrete heat equation

Tamás Szabó

In the presentation the numerical solution of the one dimensional heat conduction equation is investigated. To the discretization in space we apply the linear finite element method and for the time discretization the well-known theta-method. The aim of the talk is to derive an adequate numerical solution by this approach. We theoretically analyze the possible choice of the time-discretization step-size and establish the interval where the discrete model is reliable to the original physical phenomenon. As the discrete model, we arrive at the task of the one-step iterative method:

$$\underline{C}\frac{T^{m+1} - T^m}{\Delta t} + \underline{K}\left(\Theta T^{m+1} + (1-\Theta)T^m\right) = 0,$$

which is a system of linear algebraic equations w.r.t. the unknown vector T^{m+1} being the approximation of the temperature at the new time-level. Here the parameter θ is related to the numerical method and it is an arbitrary parameter on the interval $[0,1]$. The matrices $\underline{C}$ and $\underline{K}$ are the so-called mass and stiffness matrices, respectively. The approximation of the temperature at the

m-th time level (T^m) is assumed to be known. We apply the special cut Gauss method to obtain the solution and by the analysis of the algorithm we establish the bounds for the step-size. We point out that there is a need to obtain both lower and upper bounds of the time-step size. The main results of the work is to determine the interval for the time-step size to be used in this special finite element method and analyze the main qualitative characterstics of the model. Our theoretical results are verified different numerical experiments.

Sequential and Parallel Schemes of Decomposition of the Method of Lines for a Singularly Perturbed Parabolic Convection–Diffusion Equation with Third–Kind Boundary Conditions

I.V. Tselishcheva, G.I. Shishkin

A boundary value problem for a singularly perturbed parabolic convection–diffusion equation is considered in a rectangular domain in x and t; on the boundary of the domain, the third–kind condition admitting both Dirichlet and Neumann conditions is given. The perturbation parameter ε multiplying the highest order derivative takes arbitrary values in the half-open interval (0,1]. For the boundary value problem, we construct a scheme based on the method of lines in x passing through N_0+1 points of the mesh with respect to t. For solving the differential–difference problem on a set of intervals, we apply a domain decomposition method (on overlapping subdomains with the overlap width δ), which is a modification of the Schwarz method.

For the continual schemes of the decomposition method being constructed, we study how sequential and parallel computations, the order of priority in which the subproblems are sequentially solved on the subdomains, and the value of the parameter ε (as well as N_0, δ) influence the convergence rate of the decomposition scheme (for $N_0 \to \infty$), and also computational costs of solving the scheme and time required for its solving (unless a prescribed tolerance is achieved).

For convection–diffusion equations, in contrast to reaction-diffusion equations, the sequential scheme turns out to be more efficient than the parallel one for a large number of parallel solvers.

Decomposition Methods for Hybrid Dynamical and Hierarchical Optimization Problems

Andrey Valuev

Decomposition techniques are widely applied to mathematical programming problems having block structure, especially optimization problems for discrete-time processes. Most of them, beginning from the method by Dantzig and Wolfe, have two-level construction, i.e., combine solutions of many local problems and a coordination problem presenting the problem in question as a whole.

Two numerical methods based on solution of independent problems of finding local descent directions on each iteration without any coordination problem were proposed in 1980-s, namely block factorization method by Propoy, Krivonozhko and Tverskoy and author's method evolving Boltyansky's method of local sections. The proposed approach treats them as particular cases of a general construction of direct optimization methods based on decomposition with respect to restrictions set. A certain regular decomposition scheme gives a convenient way to represent a feasible direction as a sum of terms each of which affects a certain group of constraints and does not affect all other. Optimization algorithms construction is independent of a particular scheme. Preferences of this approach are sufficient decrease in computations amount as well as increase of computational stability.

This approach may be applied to both dynamical and quasi-dynamical optimization problems. The latter may be either traditional problems with fixed structure of variables and restrictions sets or variant structure problems emerging, e.g., for some classes of hybrid systems. For the first type, optimization problems for opencast mining must be emphasized, especially high-dimensional problems for shapes of pit contours. As to hybrid systems, recent results on optimum resources planning problems for project scheduling as well as more complicated problems for complex industrial systems combining features of "job-shop" and "flow-shop" systems are presented.

Analysis of Approximation Models and Some Computational Algorithms for Pit Contour

Andrey Valuev

Three principal types of opencast mining models, namely block, sector and contour models, are some ways of open pit shape approximation. In practice, there is only informal substantiation of likelihood between the model and its object resulting in algorithms of computational geometry for mining problems do not guaranteeing obtaining technologically possible pit shapes.

The author puts forward two principal ideas. First, the formal description of the pit shape in terms of smooth curves representing contours of each tier (lower and higher edges or the medium line of its lateral surface) with constraints on them expressing technological rules for mining. Second, two mappings must be defined, the first $p = P_{1M}(S)$ from a set of contours to the corresponding model parameters vector satisfying the model restrictions and the second $S = s_{1M}(p)$ from an admissible model parameters vector to the corresponding set of pit contours. Each model yields more or less rough representation of pit shape $S = s_{0M}(p)$. In the space of contour sets S a metric $R(S,S_1)$ is introduced based on the Hausdorff distance. So approximation error of a model type is characterized with two indices, $e_p(T)=\sup\{R(s_{0M}(p), s_{1M}(p)) \mid p \in P(T)\}$ and $e_S(T)=\sup\{R(S, s_{0M}(P_{1M}(S))) \mid S \in\Sigma(T)\}$ where $P(T)$ and $\Sigma(T)$ are respectively sets of all possible p and S for a given technological parameters vector T.

With this approach the three types of models were studied. Valuations of $e_p(T)$ and $e_S(T)$ are found showing that both block and sector models cannot decrease approximation errors lower some non-negative values. Formal definitions of contour and hybrid-type models are given for which $e_p(T)$ and $e_S(T)$ may be made arbitrary small by increasing the model dimension. Algorithms for technological problems of open pit mining are proposed that are reduced to combinations of some special primitive geometric operations. It is proved that these algorithms do not violate models restrictions.

On an Adaptive Semirefinement Multigrid Algorithm for Convection-Diffusion Problems

Daniela Vasileva

A multigrid algorithm with adaptive semirefinement is presented for the solution of convection-diffusion problems. The method is based on the discontinuous Galerkin (Baumann-Oden DG) discretisation, which can conveniently handle grid adaptation. The algorithm is presented here for 2D problems, but it can be generalized for 3D. Rectangular finite elements are used and during the process of adaptation they may be refined in the x, y or in both (x and y) directions. In order to achieve optimal efficiency, the recursive mesh adaptation is combined with a MG solver, based on Sawtooth cycles with damped block Jacobi relaxation as a smoother.

The adaptation criterion is based on the comparison of the discrete solution on the finest grid and its restrictions to the next (in the x and y directions) grids. It refines in the x or/and y direction those cells, where the corresponding difference is too large.

The numerical experiments show that the algorithm may be successfully used for resolution of boundary and interior layers. The comparison with a similar adaptive refinement multigrid algorithm shows that significantly less resources may be used for layers, almost parallel to the x or y axis.

The Numerical Spherically Symmetric Odeling of Deap-Seated Geodynamics

A. Vyatkin, V. Shaidurov and G. Shchepanovskaya

In this paper a computer model is proposed which allows one to consider geodynamics processes of the Earth's expansion, contraction, heating and cooling . Geosphere dynamics is studied in the framework of a viscous heat-conducting compressible medium where medium density and viscosity vary with time and space. This model allows one to consider not only the Earth's crust and upper mantle but the deep-seated structure including the core as well.

A Genetic-Algorithm Based Approach for the Design of Two-Channel Biorthogonal FIR Filter Banks

Felicja Wysocka-Schillak

In the paper, a new method for the design of nearly perfect-reconstruction two-channel linear-phase biorthogonal finite impulse response (FIR) filter banks is presented. The method is based on the trade-off between obtaining the equiripple reconstruction error and satisfying other additional requirements. The filter bank design problem is transformed into an equivalent multicriterion optimization problem. The obtained problem is converted into a single criterion one using the weighted sum strategy.

The paper attempts to demonstrate that a genetic algorithm (GA) can be a useful tool for solving the considered optimization problem. GAs are probabilistic search techniques based on the mechanics of natural genetics and natural selection. They have strong robustness and general utility. Because of these reasons, GAs are more likely to find optimal global solutions for multimodal and multicriterion optimization problems than classical gradient-based algorithms.

In the paper, a hybrid approach for solving the considered optimization problem is proposed. In the first step, a genetic algorithm is applied. The final point from the GA is used as the starting point for a local optimization method. The application of the GA ensures that the obtained solution is not trapped at a local minimum. Using a local optimization method in the second step results in improving the speed of convergence. Design examples are presented to illustrate the proposed technique. In these examples, linear-phase and low-delay biorthogonal filter banks with different additional requirements regarding the characteristics of the filters are considered.

Numerical Quadrature For Bessel Transformations With High Oscillations

Shuhuang Xiang

We explore higher order numerical quadrature for the integration of systems containing Bessel functions such as $\int_a^b f(x)J_m(rx)dx$ and $\int_a^b f(x)\cos(r_1x)$ $J_m(r_2x)dx$. Three families of methods are presented. One $Q_s^A(f)(s \geq 1)$ is based on a truncation of the asymptotic series, one $Q_s^L(f)$ is extending an approach in the work of Levin, and another $Q_s^E(f)$ is an extension of the work of Evans, Webster and Chung, where s is an arbitrarily given positive integer. The decay of the error of the three methods drastically improves as frequency grows:

- For $\int_a^b f(x)J_m(rx)dx$, $E(f) = O\left(\frac{1}{r^{s+3/2}}\right)$
- For $\int_a^b f(x)\cos(r_1x)J_m(r_2x)dx$, $E(f) = O\left(\frac{1}{r_2^{1/2}(\max\{r_1,r_2\})^{s+1}}\right)$.

On the Local Sensitivity of the Discrete-time H_∞ Control Problem

A. Yonchev, P. Petkov, N. Christov and M. Konstantinov

In the last 20 years, important theoretical results have been obtained in the field of H_∞ control of discrete-time systems. However, the computational aspects of the discrete-time H_∞ control problem have not been studied in a sufficient extent. For this reason, designing and implementing H_∞ controllers often lead to serious difficulties. This is due to the use of unreliable computational tools and to the lack of sensitivity estimates for the H_∞ control problem.

The paper is concerned with obtaining linear perturbation bounds for the discrete-time $\mathcal{H}_\infty$ control problem based on linear matrix inequalities (LMI). The sensitivity analysis of the perturbed matrix inequalities is considered in a similar manner as for perturbed matrix equations, after introducing a suitable right hand part, which is slightly perturbed. The proposed approach leads to tight linear perturbation bounds for the LMI solutions to the $\mathcal{H}_\infty$ control problem.

Temporally - Periodic Solutions of the Parametrically Driven Damped Nonlinear Schrödinger Equation

Elena Zemlyanaya, I. Barashenkov and N. Alexeeva

A large number of nonlinear resonant phenomena in various physical media is described by the parametrically driven damped nonlinear Schrödinger equation:

$$i\psi_t + \psi_{xx} + 2|\psi|^2\psi = h\psi^* - i\gamma\psi. \quad (1)$$

The $-i\gamma\psi$ term in the right-hand side accounts for the dissipative losses in the underlying physical system. The dissipation is compensated by pumping the energy into the system which is modelled by adding the $h\psi^*$ term to the right-hand side of Eq.(1). This particular choice of the forcing term corresponds to the parametric driving; hence the name.

Eq.(1) describes the nonlinear Faraday resonance in a vertically vibrated water trough, the parametric generation of spin waves in ferromagnets, and the phase-sensitive amplification in nonlinear optics.

We study the temporally periodic solutions of Eq.(1),

$$\psi(x, t + T) = \psi(x, t), \quad (2)$$

satisfying the vanishing boundary conditions at infinity:

$$\psi(x, t) \to 0 \quad \text{as } |x| \to \infty. \quad (3)$$

The existence, stability and bifurcation of solutions of Eqs.(1)-(3) are studied in the case of the weak, moderate and strong damping. Our analysis is based on the numerical continuation of solutions in a parameter where the starting points are furnished by the direct numerical simulation of Eq.(1). We report a variety of stable and unstable one- and two-periodic solitons. Our results shed a new light on the form of the attractor chart for Eq.(1) which has long remained a mystery.

Solving Ordinary Differential Equations By Simplex Integrals

Yongxiong Zhou, Shuhuang Xiang

This paper is devoted to the proper numerical solutions, especially to oscillating solutions, for ordinary differential equations. In contrast to Lipschitz condition, we define a new condition that

$$\left| \int_{t_0}^{t_1} f(t)dt \right| \le R \max_{\xi_1, \xi_2 \in [t_0, t_1]} |f(\xi_1) - f(\xi_2)|$$

with small R for all t_0, t_1 in the correlative intervals. Under the assumption of this condition, we obtain a new asymptotic formula

$$\phi_v(t) - Q_{v-1}(t) = O(R^v),$$

where $\phi_v(t)$ denotes the following simplex integral

$$\phi_v(t) = \int_{t_0}^{t} \cdots \int_{t_0}^{\xi_{v-2}} \int_{t_0}^{\xi_{v-1}} f(\xi_v) d\xi_v d\xi_{v-1} \cdots d\xi_1$$

and $Q_{v-1}(t)$ denotes a polynomial of degree $v-1$

$$Q_{v-1}(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \cdots + \beta_{v-1} t^{v-1},$$

in which the coefficient β_k is related to simplex integral $\phi_{n_\ell}(t)$ as above with $n_\ell > v, k, \ell = 1, 2, \ldots, v$. Then we survey the convergence, algorithms and applications to ODEs.

Characteristics of the Group Interest Network

Ning Zhang

By studying the behavior characteristics visiting world wide web of the specifically campus group, this paper constructed dynamic group interest network, which was a para-bipartite graph (see Fig 1). The time features that the group user visited world-wide-web had been observed, the topological structure had been discussed. The degree exponents had been drafted for the one week data. Although the users' visiting time is random and their surfing pages were different but the interests of a majority of the campus group were accordant. The numerical simulation indicates that the incoming degree distribution of the group interest network follows power law. And the group interest spectrum was basically steady. The visiting behavior of the campus group had their special disciplinarian.

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